Online Supplemental Appendix

Dynamic Screening in International Crises

Contents

A Proofs of Results in the Paper

A.1 Preliminaries

Before proceeding to the proofs from the main paper it is necessary to provide a full statement of the players' expected utility function. Equation (1) in the main paper was simplified for presentational purposes and did not account for the possibility that a country might concede at the same time that another country exits. The following equation addresses this issue.

$$\begin{aligned}
U_i(t_i, \theta_i, \sigma_j | w_i) = &\int_{\{w_j | t_j < t_i, \theta_j = 0\}} [f_j(w)(1 - k_i \sigma(w|0))] dw + \int_{\{w_j | t_j < t_i, \theta_j = 1\}} [f_j(w)(w_i - k_i \sigma(w|1))] dw \\
&+ \mathbb{1}_{\{t_i \neq \infty, \theta_i = 0\}} \left[\int_{\{w_j | t_j = t_i, \theta = 0\}} \left[f(w) \left(\frac{1}{2}(1 - a_i t_i) - k_i t_i \right) \right] dw + \right. \\
&\left. \int_{\{w_j | t_j = t_i, \theta = 1\}} \left[f(w) \left(\frac{1}{2}(w_i - a_i t_i) - k_i t_i \right) \right] dw - \int_{\{w_j | t_j > t_i)\}} [f_j(w)(a_i t_i + k_i t_i)] dw \right] \\
&+ \mathbb{1}_{\{t_i \neq \infty, \theta_i = 1\}} \left[\int_{\{w_j | t_j = t_i, \theta = 0\}} \left[f(w) \left(\frac{1}{2}(1 + w_i) - k_i t_i \right) \right] dw \right. \\
&\left. + \int_{\{w_j | (t_j = t_i, \theta = 1) \vee (t_j > t_i)\}} [f(w)(w_i - k_i t_i)] dw \right] - \mathbb{1}_{\{t_i = \infty\}} \int_{\{w_j | t_j = \infty)\}} [f(w)(K_i + k_i \overline{T})] dw
\end{aligned} \quad \text{(A. 1)}$$

where the first line represents the payoff if country j exits before country i, the second and third lines represent the payoffs country i can expect from conceding at time t_i, the fourth line and first term on the fifth line are the payoffs country i can expect from going to war at time t_i, and the last term is the payoff for remaining locked in the crisis forever.

Furthermore, it is necessary to provide a more formal definition of a Perfect Bayesian Equilibrium (PBE) to the game and the restrictions it places on an equilibrium. The following definition of a PBE is adapted from Takahashi (2015).

Definition A. 1 *(Perfect Bayesian Equilibrium)*
A PBE in the war of attrition consists of a pair strategies σ_i^* and σ_j^* and beliefs $g_i(w_i|t)$ that satisfy the following properties

(i) For any country i and every type w_i, σ_i^* must satisfy

$$U_i(t_i^*, \theta_i^*, \sigma_j^* | w_i) \geq U_i(\hat{t}_i, \hat{\theta}_i, \sigma_j^* | w_i)$$

*for any possible combination of $\hat{t}_i \in [0, \infty)$ and $\hat{\theta}_i \in \{0, 1\}$ for the function $U_i(\cdot)$ as defined in equation (**??**).*

(ii) $g_i(w_i|t)$ must be computed using Bayes' Rule whenever possible.

A.2 Proofs of the Lemmas

This appendix begins with a proof of the lemmas before providing a proof for each of the propositions. Though these lemmas make claims that are standard in war of attrition models, it is necessary to show that these properties still apply when states have and exercise two exit options. Lemma 2 can be applied to any interval during which neither country goes to war. Similarly, Lemma 3 can be applied to any interval of time that has only one country go to war and not the other. Lemma 4 can be applied to any interval of time during which both states go to war.

As stated in the main text, there are three possible outcomes at the horizon date: one in which both countries have finish conceding by the horizon date, one in which only one country has its unresolved types finish conceding by the horizon date and its rival has a mass of types concede on the horizon date, and a stalemate. Due to space constraints, the main text only described the first and third possibilities. However, the proofs in this supplemental appendix will address all three possibilities. To do so, I will restate Lemmas 2 and 3 and Propositions 1,3, and 6 to account for the possibility that one of the two countries can have a mass of types concede at $\overline{T}$. These will demonstrate that it is possible to have one country have a mass of types concede at the horizon date provided that its rival has not yet started to be screened by sunk costs and the game does not end in a stalemate.

A.2.1 Proof of Lemma 1

The proof of this lemma resembles the proof of lemma 1 in Fearon (1994), with minor adjustments resulting from the introduction of sunk costs. The goal is to show that countries will cease to exit in finite time. This must include two parts. The first showing that all types must concede in finite time. The second, showing that all resolved states must go to war in finite time.

First, all unresolved types must concede in finite time. Suppose not, that is suppose that for any time t', there exists a type w_i' who concedes at a time later than time t'. Recall that the cumulative audience costs that must be paid for a concession are strictly increasing. Let w_i^s denote the supremum of conceding types and $\hat{t}$ denote the time at which $a_i\hat{t}' = w_i^s$. Any type of country i conceding after time $\hat{t}$ must be strictly better off going to war instead. A contradiction. It follows that there must exist a date $\overline{T}$ after which no type of country i concedes.

Having demonstrated that all unresolved types concede in finite time, it is straightforward to demonstrate that no resolved type goes to war at a time past $\overline{T}$. Suppose not, that is suppose that type w_i went to to war at time $t > \overline{T}$. Then w_i has a strictly profitable deviation to going to war at time $t^{interim}$ for $\overline{T} < t^{interim} < t$ instead of time t and avoiding the payment of additional sunk costs.[1] It follows that there can be no strategy that has a country go to war after $\overline{T}$ that is part of an equilibrium. ■

A.2.2 Restatement of Lemma 2

Lemma A. 2 *Let* $T^1 = min\{T^p, \overline{T}\}$. *In any equilibrium* $Q_i(t)$ *must satisfy the following properties: (i)* $Q_i(t)$ *must be continuous and strictly increasing on the interval* $[0, T^1)$ *for both countries and on the interval* $[0, T^1]$ *for at least one country; (ii) if* $T^1 = \overline{T}$, *then one country can have a discontinuity in* $Q_i(\cdot)$ *at* $\overline{T}$; *(iii)* $Q_i(t) < 1$ *if and only if* $t < \min\{T^p, \overline{T}\}$; $Q_i(0)Q_j(0) = 0$.

A.2.3 Proof of Lemma 2

The proof of this lemma closely follows the proofs of similar lemmas and propositions in Fearon (1994) and Hendricks, Weiss and Wilson (1988). My proof departs from these papers in characterizing the properties of types who concede at the end of the peaceful phase $T^1 = \min\{T^p, \overline{T}\}$. In particular, the main difference is that in my lemma one country can have a mass of types concede at time $\overline{T}$.

Step 1: Types who concede during the peaceful phase must be playing a mixed strategy. To prove this we will show that the utility of any type on the interval $[0, \min\{T^p, \overline{T}\}]$ must be given by a constant that is independent of both its type and the time at which conedes. This suffices to show that the countries are indifferent and can play mixed strategies represented by the cumulative distribution function $Q_i(t)$.

Suppose not, that is, suppose that there were two different types w_i^1 and w_i^2 who conceded on the interval $[0, \min\{T^p, \overline{T}\}]$ and had different expected utilities. Without loss of generality, assume that $U_i(\cdot|w_i^1) > U_i(\cdot|w_i^2)$. First, observe that this cannot be possible if the two types concede at the same time as the types face no risk of war by assumption. However, if the two types concede at different times, then type w_i^2 has a profitable deviation by switching to the strategy played by type w_i^1. But the two types would then have the same expected utility. A contradiction.

Step 2: $Q_i(0)Q_j(0) = 0$: Suppose not, that is suppose that $Q_i(0), Q_j(0) > 0$. This implies that the utility of country i if it concedes at time 0 is given by $U_i(0,0) = \frac{Q_j(0)}{2}$. However, if country i delayed concession by some arbitarily small ϵ, then it could strictly increase its utility to $U_i(\epsilon, 0) = Q_j(\epsilon) - \epsilon a_i$ which for ϵ approaching zero is equal to $Q_j(0)$.[2] A contradiction.

Step 3: $Q_i(t) < 1$ **if and only if** $t < \min\{T^p, \overline{T}\}$**.** Let $T^1 = \min\{T^p, \overline{T}\}$. The proof of this claim has two steps. First, it is necessary to show that it cannot be the case that both $Q_i(t) = 1$ and $Q_j(t) = 1$ for some $t < T^1$. To prove this, suppose not such that no type concedes on the

[1]This is true regardless of whether it is assumed that type w_i is paying $k_i(t - \overline{T})$ sunk costs or $\overline{K}_i$ sunk costs.

[2]Because $Q_i(t)$ is an increasing function on a compact interval it must be continuous almost everywhere. It follows that there must exist an ϵ small enough such that there is no mass point of types j conceding on the interval $(0, \epsilon]$ thereby implying that the probability of a concession by j on that interval is effectively zero for sufficiently small ϵ.

interval $[t, T^1]$. Recall that, by definition, there must be at least one type of country j who goes to war at time T^1. This type of country j could however, strictly increase its utility by instead going to war at time t and paying less sunk costs, thereby implying that the peaceful phase would end at time t and not time T^1.

Second, it is necessary to show that that it cannot be the case that $Q_i(t) = 1$ for some $t < T^1$ while $Q_j(t) < 1$ for all $t < T^1$ and $Q_j(T^1) = 1$. To prove this, suppose not; i.e. suppose that the statement were true. In this case, any type of country i conceding at some time $t' \in [t, T^1]$ has a strictly profitable deviation to instead conceding at time $t' - \epsilon$ for some arbitrarily small $\epsilon > 0$. This is because country i is not conceding during the interval $[t' - \epsilon, t']$ and country j can pay less sunk costs and audience costs by conceding earlier. A contradiction.

Step 4: $\lim\limits_{t \to \hat{t}} Q_i(\hat{t}) = Q_i(t)$ **for** $t \in (0, T^1)$**.** In other words, $Q_i(t)$ can have no mass points on the interval $(0, T^1)$. Suppose not and assume that country i has a mass of types conceding at time $t \in (0, T^1)$. Then any type of country j conceding on the interval $[t - \epsilon, t]$ can strictly increase their utility by conceding slightly after time t at time $t + \epsilon$.

$$U_j(t+\epsilon) - U_j(t-\epsilon) = \int_{t-\epsilon}^{t+\epsilon} [q_i(t)(1 - k_j(t))]dt$$
$$-[1 - Q_i(t+\epsilon)][a_j(t+\epsilon) + k_j(t+\epsilon)] + [1 - Q_i(t-\epsilon)][a_j(t-\epsilon) + k_j(t-\epsilon)]$$

which when taking the limit of ϵ to zero leaves

$$q_i(t) > 0$$

This implies that there exists some $\epsilon > 0$ such that no type of country j will concede on the interval $[t - \epsilon, t + \epsilon]$. However, the mass of types of country i conceding at time t have a strictly dominant deviation to conceding at time $t - \epsilon$ and paying less sunk costs and audience costs. A contradiction.

Step 5: $Q_i(t') \neq Q_j(t'')$ **for any** $t' \neq t''$**.** In other words there can be no interval during which unresolved types that concede during the peaceful phase do not exit with positive probability. Suppose not. That is suppose, that there existed an interval $[t', t'']$ during which Country j did not exit. Note that it cannot be the case that $t'' = T^1 = \min\{T^p, \overline{T}\}$ as this would violate Step 3. Therefore $t'' < T^1$. Note that Country j would never concede on the interval $(t', t'']$, since they could instead concede at time t' and avoid paying additional sunk costs and audience costs. In addition, there exists an arbitrarily small ϵ such that any type of Country i that concedes on some interval $[t'', t'' + \epsilon]$ could strictly increase its utility by conceding at time t' instead. To see this observe that

$$U_i(t') - U_i(t'' + \epsilon) = \int_{t''}^{t''+\epsilon} q_j(t)[1 - k_i t]dt - k_i[t'' + \epsilon - t']$$

which equals $k_i(t'' - t')$ when taking the limit of $\epsilon \to 0$. This implies that Country i does not concede on the interval $[t', t'' + \epsilon]$. However, in this case, any type of Country j conceding on the interval $[t'', t'' + \epsilon]$ could strictly increase its utility by instead conceding at time t' and avoid paying the additional sunk costs and audience costs. This contradicts our original premise that the interval of time on which Country i did not concede was $[t', t'']$.

Step 6: $\lim_{t \to \hat{T}^p} Q_i(\hat{t}) = Q_i(T^p)$ **for** $T^1 = T^p < \overline{T}$**.** In other words, $Q_i(t)$ can have no mass points at T^p when the game transitions to the first screening phase (or in the case in which $T_1^p = T_2^p$, the second screening phase).[3] Suppose not. That is, suppose that there were a mass of types of country i who conceded at time T^p. Following Lemma 3, we know that an unresolved type of country j conceding at some point $T^p - \epsilon$ during the peaceful phase could increase their expected utility by instead conceding at some point $T^p + \epsilon$ instead

$$U_j(T^p + \epsilon, 0; w_j) - U_j(T^p - \epsilon, 0; w_j) = F_i(\beta_i^p) \int_{T^p - \epsilon}^{T^p} q_i(t)[1 - k_j]dt$$
$$+ \int_{T^p}^{t_j} f_i(\tau_i(t, 0))\tau_i'(t, 0)[1 - k_j t]dt - (a_j t_j - k_j t_j)[1 - F_i(\tau(t_j, 0))]$$

Taking the limit of ϵ to 0, we find that this equation equals $q_i(T^p) > 0$. Since step 1 of this proof implies that all types conceding during the peaceful phase must have the same expected utility, any type of country j conceding during the first screening phase must strictly prefer to instead concede at some time $T^p + \epsilon$. But then, if no type of country j concedes during the first screening phase, then the mass of types of country i that concede at time T^p could strictly increase their expected utility by conceding instead at time 0 and not paying any sunk costs. A contradiction. The proof to demonstrate that Country j does not have a mass of types conceding at time T^1 follows identical steps.

Step 7: If $T^1 = \overline{T}$**, then** $\lim_{t \to \widehat{T}} Q_i(\widehat{T})$ **may be greater than** $Q_i(\hat{t})$ **for one country.** First, if Country i has a mass of types concede at time t, then no type of Country j will be willing to fight at $\overline{T}$ since they could strictly increase their utility from $\frac{F_i(Beta_i)q(\overline{T})}{2} + (1 - F_i(Beta_i))w_j$ to $F_i(Beta_i)q(\overline{T}) + (1 - F_i(Beta_i))w_j$ by delaying their decision to exit and seeing whether their rival will go to war or concede. Note that Country j's strategy would not violate Lemma 1 if and only if all types of Country i exit by $\overline{T}$. Therefore, Country i can only have a mass point at time $\overline{T}$ if all types exit at $\overline{T}$.

Second, only one country can have a measurable mass of types concede at time $\overline{T}$. If both countries had a mass of types concede at time $\overline{T}$, then following a similar logic to the above argument, neither country would go to war at that time. However, if neither country conceded at time $\overline{T}$ then both countries would seek to go to war after that time, a violation of Lemma 1. ■

A.2.4 Restatement of Lemma 3

Lemma A. 3 *Let* $T^2 = \min\{T^f, \overline{T}\}$*. If there exists a* $T^p < \overline{T}$*, then in any equilibrium* $S_j(t)$ *must satisfy the following properties: (i)* $\sigma_i(\cdot|0)$ *must be continuous and strictly increasing on* $[T^p, T^2]$ *(ii)*

[3] Note that this proof relies on the properties stated in Lemma 3 (or Lemma 4 if the game proceeds from the peaceful phase to the second screening phase). However, the proof of the properties of Lemma 3 (and Lemma 4) do not rely on this proof.

$S_j(t)$ and $\sigma_i(\cdot|0)$ must be continuous and strictly increasing on the interval $[0, T^2)$; (iii) if $T^2 = \overline{T}$, then $S_j(\cdot)$ may have a discontinuity at $\overline{T}$; (iv) $S_j(t) < 1$ if and only if $t < \min\{T^f, \overline{T}\}$; (v) $\sigma_j(\cdot|1)$ must be continuous and strictly decreasing on $[T^p, T^2]$.

A.2.5 Proof of Lemma 3

Step 1: Types of country j that concede during the peaceful phase must be playing a mixed strategy. Follows identical steps to the proof of Lemma 2, step 1.

Step 1: $S_i < 1$ if and only if $t < \min\{T^f, \overline{T}\}$. The proof of this claim follows analogous arguments to Lemma 2, Step 3.

Step 2: $S_j(t) < 1$ if and only if $t < T^2$. Suppose not. That is, suppose that $S_j(t') = 1$ for some $t' < \overline{T}$. This implies that country j does not concede on the interval $[t', T^2]$. Recall that there must be at least one type of country i that fights at time T^2 by definition of T^f and $\overline{T}$. This type has a strictly profitable to fight instead at time t' and avoid paying sunk costs. A contradiction.

Step 3: Country i cannot have a mass of types concede at any time. Suppose not. That is, suppose that there is a mass of types of country i conceding at time t where $T^p \leq t < \min\{T^f, \overline{T}\}$. It follows that there exists some interval $[t - \epsilon, t]$ in which country j never exits.

$$U_j(t+\epsilon, \theta; w_j) - U_j(t-\epsilon, \theta; w_j) = \int_{t-\epsilon}^{t+\epsilon} \int_{\{w_i \in \tau_i(t,0)\}} f_i(w)[1 - k_j t] dw dt$$
$$+ \int_{\{w_i \in \tau_i(t_i, \theta | t_i > t+\epsilon)\}} f_i(w)[\mathbb{1}_{\{\theta=1, t_i \neq \infty\}} w_i - \mathbb{1}_{\{\theta=0, t_i \neq \infty\}} a_i(t+\epsilon) - k_j(t+\epsilon)] dw$$
$$- \int_{\{w_i \in \tau_i(t_i, \theta | t_i > t-\epsilon)\}} f_i(w)[\mathbb{1}_{\{\theta=1, t_i \neq \infty\}} w_i - \mathbb{1}_{\{\theta=0, t_i \neq \infty\}} a_i(t-\epsilon) - k_j(t-\epsilon)] dw$$

Or taking the limit of ϵ to zero

$$\int_{\{w_i \in \tau_i(t,0)\}} f_i(w)[1 - \mathbb{1}_{\{\theta=1, t_i \neq \infty\}} w_i - \mathbb{1}_{\{\theta=0, t_i \neq \infty\}} a_i t] dw > 0$$

This implies that no type of country j concedes in the interval $[t-\epsilon, t]$. It then follows that country i only incurs sunk costs by delaying its concession time from $t-\epsilon$ to time t and could strictly increase its payoff by conceding at time $t-\epsilon$ instead. A contradiction.

Step 4: There can be no interval $[t', t'']$ during which no type of country i does not concede. Suppose not. That is, suppose that there is an interval $[t', t'']$ during which no type of country i concedes and where $T^p \leq t' < t'' \leq \min\{T^f, \overline{T}\}$. This implies that there exists an ϵ such that no type of country j exits during the interval $(t', t''+\epsilon]$ as they could instead exit at time t' and avoid paying sunk costs. The fact that there is no point in time at which a mass of types of country i concedes, ensures that there can be no benefit to country j from delaying their exit until

$t'' + \epsilon$ as the following equation shows

$$U_j(t', \theta; w_j) - U_i(t'' + \epsilon, \theta; w_j) = -\int_{t''}^{t''+\epsilon} \int_{\{w_i \in \tau_i(t,0)\}} f_i(w)[1 - k_j t] dw dt$$
$$+ \int_{\{w_i \in \tau_i(t_i, \theta | t_i > t')\}} f_i(w)[\mathbb{1}_{\{\theta=1, t_i \neq \infty\}} w_i - \mathbb{1}_{\{\theta=0, t_i \neq \infty\}} a_j t' - k_j t'] dw$$
$$- \int_{\{w_i \in \tau_i(t_i, \theta | t_i > t'')\}} f(w)[\mathbb{1}_{\{\theta=1, t_i \neq \infty\}} w_i - \mathbb{1}_{\{\theta=0, t_i \neq \infty\}} a_j (t'' + \epsilon) - k_j (t'' + \epsilon)] dw$$

which taking the limit of ϵ to zero leaves

$$k_i t'' - k_i t' + \mathbb{1}_{\{\theta=0, t_i \neq \infty\}} a_i t'' - \mathbb{1}_{\{\theta=0, t_i \neq \infty\}} a_i t' > 0$$

This implies that no type of country j ever exits in the interval $[t'', t'' + \epsilon]$. However, if no type of country j exits in the interval country $[t'', t'' + \epsilon]$, then the types of country i exiting in the interval $[t'', t'' + \epsilon]$ can strictly increase their payoff by conceding at time t' instead. This contradicts the premise that the interval of time during which country i does not exit is $[t', t'']$.

Step 5: $\sigma_j(\cdot|1)$ is continuous and strictly decreasing. Suppose that there is a mass of types of country j exiting at time t where $T^p \leq t < \min\{T^f, \overline{T}\}$. Given steps 1 and 2, there must exist a function $Z_i(t)$ that describes the probability that country i has conceded during the first screening phase that is both continuous and strictly increasing. Without loss of generality, assume for the purposes of this step that $\sigma_i(\cdot|0)$ is continuous and strictly increasing. This simplifies the notation and ensures that country j's expected utility can be represented by equation (A. 18).

Country j's utility function is continuous in both t and w_j. This implies that any type of country j choosing to go to war at time t must have its utility function satisfy

$$\frac{\partial U_j(t, 1; w_j)}{\partial t} = f_i(\tau_i(t, 0))\tau_i'(t, 0)[1 - w_j] - k_j[1 - F_j(\tau(t, 0)] = 0$$

It is straightforward to see that any type less resolved than w_j also exiting at time t would have a strictly positive benefit to waiting and exiting later. Moreover, any type more resolved than w_j would have a strictly positive benefit to exiting earlier. This contradicts the premise that there is a mass exiting at time t.

Alternatively suppose that there were an interval $[t', t'']$ during which no type of country j chose to fight. It is still without loss of generality to assume that country j's utility function would be given by equation (A. 18) which is continuous in both w_j and t. A type w_j' would only exit at time t' if $\frac{\partial U_j(t', 1; w_j)}{\partial t'} = 0$. Similarly a type w_j'' would only exit at time t'' whom $\frac{\partial U_j(t'', 1; w_j'')}{\partial t''} = 0$. From the continuity of country j's utility in t and w_j, there must be a type $w_j \in (w_j'', w_j')$ who strictly prefers to exit in the interval (t', t''), a contradiction.

To complete the proof it is sufficient to show that the equation (A. 18) satisfies single crossing. Taking the derivative with respect to t and w_j we are left with $-f_i(\tau_i(t, 0))\tau_i'(t, 0)$ which implies that it country j's strategy must be strictly decreasing (Ashworth and Bueno de Mesquita 2006).

Step 6: $\lim_{t\to\hat{t}} S_j(t) = S_j(\hat{t})$ for $\hat{t} \in [T^p, T^2)$. In other words, country j can have no mass of types conceding on the interval $[T^p, T^2)$. Suppose not. That is, suppose that country j had a mass of types conceding at time $t \in (T^p, T^2)$. Then there exists an $\epsilon > 0$ such that any type of country i conceding on the interval $[t-\epsilon, t]$ would strictly benefit by delaying their concession to time $(t, t+\epsilon]$

$$U_i(t+\epsilon, 0|w) - U_i(t-\epsilon, 0|w) = \int_{t-\epsilon}^{t+\epsilon} [s_j(t)(1-k_it - f_j(\tau_j(t,1))\tau_j'(t,1)(w_i - k_it)dt$$
$$-[F_j(\tau_j(t+\epsilon)) - [F_j(\beta_j^f) - F_j(\beta_j^p)(1 - S_j(t+\epsilon)) + F_j(\beta_j^p)][a_i(t+\epsilon) + k_i(t+\epsilon)]$$
$$+[F_j(\tau_j(t-\epsilon)) - [F_j(\beta_j^f) - F_j(\beta_j^p)(1 - S_j(t-\epsilon)) + F_j(\beta_j^p)][a_i(t-\epsilon) + k_i(t-\epsilon)]$$

which, when taking the limit of ϵ to zero, must equal

$$s_j(t)[1 - k_it]$$

which is strictly greater than zero. However, such a deviation by country i would violate Step 4 of this proof. Therefore, country j cannot have a mass of types conceding on the interval $[T^p, T^2)$.

Step 7: $S_j(t') \neq S_j(t'')$ for any $t' \neq t''$. In other words there can be no interval during which country j does not exit. Suppose not. That is suppose that there existed a non-degenerate interval $[t', t'']$ during which country j did not concede so that $S_j(t') = S_j(t'')$. Then there exists an $\epsilon > 0$ such that any type conceding on the interval $[t', t'' + \epsilon]$ would strictly prefer to concede at time t' instead. To see this note that

$$U_i(t', 0|w_i) - U_i(t'' + \epsilon|w_i) = -\int_{t'}^{t''+\epsilon} [s_j(t)(1 - k_it) - f_j(\tau_j(t,1))\tau_j'(t,1)(w_i - k_it)]dt$$
$$-[F_j(\tau_j(t')) - (F_j(\beta_j^f) - F_j(\beta_j^p))(1 - S_j(t')) - F_j(\beta_j^p)][1 - S_j(t)][a_i(t') + k_i(t')]$$
$$+[F_j(\tau_j(t''+\epsilon)) - (F_j(\beta_j^f) - F_j(\beta_j^p))(1 - S_j(t''+\epsilon)) - F_j(\beta_j^p)][1 - S_j(t)][a_i(t''+\epsilon) + k_i(t''+\epsilon)]$$

which when taking the limit of ϵ to 0, leaves

$$\int_{t'}^{t''} [f_j(\tau_j(t,1))\tau_j'(t,1)(w_i - k_it)]dt + [(F_j(\beta_j^f) - F_j(\beta_j^p))(1 - S_j(t')) - F_j(\beta_j^p)](a_i + k_i)(t'' - t')$$
$$+F_j(\tau_j(t'',1))(a_it'' + k_it'') - F_j(\tau_j(t',1))(a_it' + k_it')$$

all these terms are positive, thereby implying that no type of country i concedes on the interval $[t'', t'' + \epsilon]$ for some small positive $\epsilon > 0$. However, in this instance, any type of country j conceding on the interval $[t'', t + \epsilon'']$ could instead concede at time t' and avoid paying additional sunk costs. But we assumed at the start the interval during which country j did not concede was $[t', t'']$. A contradiction.

Step 8: If $T^2 = \overline{T}$, then $\lim_{t\to\hat{T}} S_j(t)$ may be smaller than $Q_i(\overline{T})$. That is, country j may have a mass of types concede at time $\overline{T}$ so that $S_j(t)$ may have a discontinuity at time $\overline{T}$. The proof of this step follows an identical logic to that in Lemma 2, Step 7.[4]

Step 9: $\sigma_i(\cdot|0)$ is strictly increasing. To prove this claim, it is sufficient to show that country i's expected utility function satisfies the single crossing property. First, note that in light of steps 3, 4, 5, and 6, country i's expected utility function can be represented with

$$U_i(t_i, 0; w_i|T^p \leq t_i \leq T^f) = F_j(\beta_j^p) \int_0^{T^p} q_j(t)[1 - k_i t]dt \tag{A. 2}$$

$$+[F_j(\beta_j^f) - F_j(\beta_j^p)] \int_{T^p}^{t_i} s_j(t)[1 - k_i t]dt - \int_{T^p}^{t_i} f_j(\tau_j(t, 1))\tau_j'(t, 1)[w_i - k_i t]dt \tag{A. 3}$$

$$-[F_j(\tau_j(t_i, 1)) - (F_j(\beta_j^f) - F_j(\beta_j^p))S_j(t_i) - F_j(\beta_j^p)][a_i t_i + k_i t_i] \tag{A. 4}$$

and is continuous in both t_i and w_i. The cross-partial of country i's utility with respect to t_i and w_i is $-f_j(\tau_j(t_i, 1)\tau_j(t_i, 1)$ which is positive. This is sufficient to show that $\sigma_i(\cdot|0)$ is strictly increasing (Ashworth and Bueno de Mesquita 2006). ■

A.2.6 Proof of Lemma 4

Step 1: Country i ($i = 1, 2$) can have a mass of types conceding at time t if it also has a mass of types escalating at time t. Suppose not. That, is, suppose that there were a time t where a mass of types of country j conceded but there were no mass of types of country j escalating. In that case, there exists an $\epsilon > 0$ such that any types of country i exiting during the interval $[t - \epsilon, t + \epsilon)$ could strictly increase their payoff by conceding at time $t + \epsilon$

$$\begin{aligned} U_i(t + \epsilon, \theta; w_i) - U_i(t - \epsilon, \theta; w_i) = &\int_{t-\epsilon}^{t+\epsilon} \int_{\{w_j \in \tau_j(t,0)\}} f_j(w)[1 - k_i t)]dw \\ &+ \int_{\{w_j \in \tau_j(t,1)\}} f_j(w)[w_i - k_i t]dwdt \\ + \int_{\{w_j \in \tau(t_j, \theta|t_j > t+\epsilon)\}} &f_j(w)[\mathbb{1}_{\{\theta=0, t_i \neq \infty\}} w_i - \mathbb{1}_{\{\theta=0, t_i \neq \infty\}} a_i(t + \epsilon) - k_j(t + \epsilon)]dw \\ - \int_{\{w_j \in \tau(t_j, \theta|t_j > t-\epsilon)\}} &f_j(w)[\mathbb{1}_{\{\theta=0, t_i \neq \infty\}} w_i - \mathbb{1}_{\{\theta=0, t_i \neq \infty\}} a_i(t - \epsilon) - k_j(t - \epsilon)]dw \end{aligned}$$

Taking the limit of ϵ to 0, we find that we are left with

$$\int_{\{w_j \in \tau_j(t,0)\}} f_j(w)[1 - k_i t]dw > 0$$

This implies that no type of country i concedes during the interval $[t - \epsilon, t + \epsilon]$. But then any type of country j conceding at time t could strictly strictly increase their payoff by conceding at time $t - \epsilon$ and avoid avoid paying sunk costs. A contradiction.

Step 2: If country j has a mass of types concede at time t, then there exists an $\epsilon > 0$

[4] The sole difference is that we have already ruled out Country i having a mass of types concede at time $\overline{T}$ with Step 3 of this proof.

such that country i does not go to war during the interval $[t-\epsilon, t]$. Suppose not. That is, suppose that country j had a mass of types conceding at time t and that for every $\epsilon > 0$ there were always some type of country i who went to war in the interval $[t-\epsilon, t]$. That type of country i could strictly increase its payoff by conceding at time $t+\epsilon$ as

$$U_i(t+\epsilon, 1; w_i) - U_i(t-\epsilon, 1; w_i) = \int_{t-\epsilon}^{t+\epsilon} \int_{\{w_j \in \tau_j(t,0)\}} f_j(w)[1-k_i t]dw + \int_{\{w_j \in \tau_j(t,1)\}} f_j(w)[w_i - k_i t]dwdt$$

$$+ \int_{\{w_j \in \tau(t_j,\theta|t_j > t+\epsilon)\}} f_j(w)[w_i - k_j(t+\epsilon)]dw - \int_{\{w_j \in \tau(t_j\theta|t_j > t-\epsilon)\}} f_j(w)[w_i - k_i(t-\epsilon)]dw$$

which when taking the limit of ϵ to zero leaves

$$\int_{\{w_j \in \tau_j(t,0)\}} f_j(w)[1-w_i]dw > 0$$

This shows that country i can strictly increase its payoff by going to war at time $t+\epsilon$ instead of time $t-\epsilon$. A contradiction.

Note that Step 2 also implies that both countries cannot have a mass of types concede at the same time t, as step 1 implies that this would also require that both countries have a mass of types go to war at time t, thereby contradicting the previous result.

Step 3: Suppose country j has a mass of types conceding at time t. Then there can be no interval $[t-\epsilon, t)$ during which no type of either country concedes. Suppose not. That is, suppose that there were some $\epsilon > 0$ such that no type of country i conceded in the interval $[t-\epsilon, t)$. Step 2 establishes that there must be some interval $[t-\epsilon', t]$ during which country i does not go to war. Following the arguments in the main text, types of country j must be indifferent as to when they concede during the interval $[t-\epsilon, t)$. If there were an interval $[t-\epsilon, t)$ such that country i did not concede during that interval, then types of country j conceding at time t would have a profitable deviation to conceding at time $t-\epsilon$ and avoid paying sunk costs. A contradiction.

Alternatively, there can be no interval $[t-\epsilon, t)$ during which no type of country j concedes. If there were such an interval, then any type of country i conceding in that interval could simply concede at time $t-\epsilon$ and avoid paying sunk costs and the threat of war. This contradicts the previous result.

Step 4: Suppose country j has a mass of types conceding at time t'. Then there can be no interval $[t', t'']$ during which no type of either country i or country j does not concede. Suppose not. That is, suppose that there were an interval $[t', t'']$ during which no type of either country i or country j did not concede. Without loss of generality assume that it is country i that has no types concede during $[t', t'']$. Step 3 implies that there can be no mass point of conceding types at time t'' (by either country). It follows that any type of country j exiting during the interval $(t', t''+\epsilon]$ for some $\epsilon > 0$ could strictly increase their payoff by exiting at time t' and avoid paying sunk costs. But then any type of country i conceding at time t'' could strictly increase their payoff by conceding at time t'. A contradiction.

Step 5: There can be no mass of countries conceding. Suppose not. That is, suppose that country j had a mass of types conceding at time t. It follows from the previous steps that there must exist some cumulative distribution function $Z_i(t)$ that is continuous and strictly increasing describing the probability that country i concedes in the neighborhood of time t. This implies that the utility for a type of country j that escalates at time t is continuous on some interval $[t-\epsilon, t+\epsilon]$. Therefore a type of country j going to war at time j must satisfy $\frac{\partial U_j(t,1;w_j)}{\partial t} = 0$. However, this will only be true for country j when

$$\frac{z_i(t)}{1-Z_i(t)} = \frac{k_j}{1-w_j}$$

If this is satisfied for some type w_j escalating at time t, then any type less resolved and also escalating at time t that could strictly increase its expected utility by escalating later in that interval. Similarly, any type more resolved than w_j could strictly increase its expected utility by escalating prior to time t. This contradicts the requirement that there is a mass of types of country j going to war at time t established in Step 1. Therefore, there can therefore be no mass of types of country j conceding at time t.

Step 6: There can be no interval $[t', t'']$ during which no type of country i ($i = 1, 2$) does not concede. The proof of this claim follows identical steps to that in Lemma 3, Step 7.

Step 7: $\sigma_i(\cdot|1)$ is strictly decreasing (for $i = 1, 2$) Steps 5 and 6 imply that there must exist some continuous and strictly increasing cumulative distribution function $C_j(t)$ representing the probability that country j concedes during the second screening phase. The arguments presented in Step 5 of this Lemma can therefore be extended to rule out the possibility of a mass point of types of country i going to war at any time (as opposed to in conjunction with a mass of types of country i conceding). It is then possible to replicate the arguments made in Lemma 3, Step 5 to show that there there can be no interval $[t', t'']$ where no type of country i ($i = 1, 2$) goes to war. To complete the step, it is only necessary to show that country i's utility satisfies single crossing. As in Lemma 3, Step 5, the cross-partial of country i's utility function with respect to its exit time t_i and its type w_i is $-z_i(t)$ which is negative as desired.

Step 8: $\sigma_i(\cdot|0)$ is strictly increasing As in Lemma 3, step 9, it is sufficient to show that country i's utility satisfies single crossing. Using the previous steps we can rewrite the expected utility of a type of country i conceding during the first screening phase as

$$\begin{aligned} U(t_i, \theta; w_i | T^f \leq t_i < \overline{T}) = F_j(\beta_j^p) \int_0^{T^P} q_j(t)[1 - k_i t]dt + [F_j(\beta_j^f) - F_j(\beta_j^p)] \int_{T^p}^{T^f} s_j(t)[1 - k_i(t)]dt \\ + \int_{T^f}^{t_i} f_j(\tau_j(t, 0))\tau_j'(t, 0)[1 - k_i(t)]dt - \int_{T^p}^{t_i} (\tau_j(t, 1))\tau_j'(t, 1)[w_i - k_i t]dt \\ + [F_j(\tau_j(t_i, 1)) - F_j(\tau_j(t_i, 0))] \max\{-a_i t - k_i t, w_i - k_i t\} \end{aligned}$$

Taking the cross partial of this expected utility function, we are left with $-f_j(\tau_i(t_i,1))\tau_j'(t_i,1)$ which is positive as desired. ■

A.3 Proofs of the Propositions

To prove the propositions it is necessary to show that no type has a profitable deviation from its prescribed strategy. This requires checking that no type can increase its expected utility by changing its exit time and its exit option. In the proofs of Propositions 1-5, I hold countries exit option constant and only rule out deviations from prescribed exit times for their assigned phases. Deviating from prescribed exit times to those in different phases is easily ruled out because of the monotonicity of the pure strategies during the screening phases. I do not restate this argument for each individual proposition. Proposition 6 verifies the assignment of types to their exit strategies.

It is easiest to prove the propositions 3 and 4 that characterize strategies in the first screening phase after proving propositions 1,2 and 5, which characterize behavior in the other phases. Proposition 6 is proven last.

A.3.1 Restatement of Proposition 1

Proposition A. 1 *Let* $T^1 = \min\{T^p, \overline{T}\}$. *If* $T^1 = T^p$, *then types* $w_i \in [\underline{w}_i, \beta_i^p]$ $(i = 1, 2)$ *concede on the interval* $[0, T^1]$ *according to the following strategy*

$$\frac{q_j(t)F_j(\beta_j^p)}{1 - F_j(\beta_j^p)Q_j(t)} = \frac{a_i + k_i}{1 + a_i t} \tag{A. 5}$$

If $T^1 = \overline{T}$, *then one country may instead play according to equation (A. 5) on the interval* $[0, \overline{T})$ *and play*

$$q(\overline{T}) = \lim_{t \to \overline{T}^-} 1 - Q(t) \tag{A. 6}$$

Since no type goes to war during the peaceful phase, both countries beliefs are given by

$$g_i(w_j|t) = \begin{cases} \frac{f_j(w_j)[1-Q_j(t_i)]}{1-Q_j(t)F_j(\beta_j^p)} & \text{if } w_j \in [\underline{w}_j^t, \beta_j^p] \\ \frac{f_j(w_j)}{1-Q_j(t)F_j(\beta_j^p)} & \text{if } w_j \in [\beta_j^p, \overline{w}_j] \end{cases} \tag{A. 7}$$

A.3.2 Proof of Proposition 1

Types who concede during the peaceful phase play a mixed strategy and are indifferent as to when they concede. Therefore, they cannot profitably deviate to playing any other strategy that would have them concede during the peaceful phase. ■

A.3.3 Proof of Proposition 2

Proposition 1 implies that I can restate the utility function in equation (A. 1) for a country going to war during the peaceful phase as

$$U_i(t_i, 1; w_i | t \leq T^p) = F_j(\beta_j^p) \int_0^{t_i} q_j(t)[1 - k_i(t)]dt + [1 - F_j(\beta_j^p) Q_j(t_i)][w_i - k_i t_i] \tag{A. 8}$$

Time T_i^p is derived by taking the first-order condition of this expected utility function. Therefore, to prove that no type of any country will go to war before T^p, it is necessary to show that equation (A. 8) is concave in t_i. This can be verified by showing that the second order condition is negative. Taking the derivative of equation (A. 8) with respect to t_i twice, we find that it is is concave if

$$F_j(\beta_j^p) \frac{q_j(t)}{dt}[1 - \overline{w}_i] + k_i F_j(\beta_j^p) q_j(t_i) < 0$$

To find the value of $F_j(\beta_j^p) \frac{q_j(t)}{dt}$, we can rearrange the expression in equation (A. 5) to find that

$$F_j(\beta_j^p) q_j(t) = \frac{a_i + k_i}{1 + a_i t}[1 - F_j(\beta_j^p) Q_j(t)]$$

and then take the derivative with respect to t

$$F_j(\beta_j^p) \frac{dq_j(t)}{dt} = -\frac{a_i[a_i + k_i]}{[1 + a_i t]^2}[1 - F_j(\beta_j^p) Q_j(t)] - F_j(\beta_j^p) q_j(t) \frac{a_i + k_i}{1 + a_i t}$$

Substituting this back in to the second order condition, we are left with

$$\left[-\frac{a_i[a_i + k_i]}{[1 + a_i t_i]^2}[1 - F_j(\beta_j^p) Q_j(t_i)] - F_j(\beta_j^p) q_j(t_i) \frac{a_i + k_i}{1 + a_i t_i} \right] [1 - \overline{w}_i] + k_i F_j(\beta_j^p) q_j(t_i) < 0$$

I begin by dividing by $[1 - F_j(\beta_j^p) Q_j(t_i)]$ and multiplying by $[1 + a_i t_i]^2$ to get

$$\left[-a_i[a_i + k_i] - \frac{F_j(\beta_j^p) q_j(t)}{1 - F_j(\beta_j^p) Q_j(t)}[a_i + k_i)][1 + a_i t_i] \right] [1 - \overline{w}_i] + k_i \frac{F_j(\beta_j^p) q_j(t_i)}{1 - F_j(\beta_j^p) Q_j(t_i)}[1 + a_i t_i]^2 < 0$$

Substituting in for the hazard rates, we have

$$\left[-a_i[a_i + k_i] - [a_i + k_i]^2 \right] [1 - \overline{w}_i] + k_i [a_i + k_i][1 + a_i t_i] < 0$$

We divide by $[a_i + k_i]$ to get

$$-\left[2a_i + k_i\right] [1 - \overline{w}_i] + k_i [1 + a_i t_i] < 0$$

Isolating $\overline{w}_i$ we have

$$-\left[2a_i + k_i\right] + k_i [1 + a_i t_i] < -\overline{w}_i [2a_i + k_i]$$

Or dividing by $-[2a_i + k_i]$ we have

$$\overline{w}_i < 1 - \frac{k_i}{2a_i + k_i}[1 + a_i(t_i)]$$

From equation (4), we know that

$$\overline{w}_i = 1 - \frac{k_i}{a_i + k_i}[1 + a_i t_i]$$

which is indeed less than the term on the right-hand side. ■

A.3.4 Proof of Proposition 5

To prove the proposition, we need to verify that $\sigma_i(\cdot|1)$ is strictly decreasing in the second screening phase and to show that each country's expected utility function is concave in t_i. If this is the case, then the strategies in equations (12) and (13) that were derived from first order conditions must be utility maximizing.

To start, we will verify that the expression in (13) is negative. The denominator in that expression

$$[1 - \overline{w}_i][\underline{w}_i + a_i t]$$

is negative. Therefore for the expression to negative as required, the numerator must be positive. The numerator will be positive if

$$k_i[\overline{w}_i + a_i t] - a_i[1 - \overline{w}_i^t] > 0$$

which can be rearranged to

$$\overline{w}_i > 1 - \frac{k_i}{a_i + k_i}[1 + a_i t]$$

To see that this holds, observe that we can rearrange the expression in (13) to show that $\overline{w}_i^t$ is given by

$$\overline{w}_i^t = 1 - \frac{k_i t[1 + a_i t]}{a_i + k_i \frac{f_j(\tau_j(t,1))}{F_j(\tau_j(t,1)) - F_j(\tau_j(t,0))}[\underline{w}_i^t + a_i t]}$$

Substituting in this value for $\overline{w}_i^t$ back into the inequality, it is straightforward to see that the numerator is positive.

Next, we examine the second order condition for a type that is going to war. Using Lemma 4, we can rewrite the utility function for types of country i ($i = 1, 2$) who exit during the second screening phase as follows

$$U(t_i,\theta;w_i|T^f \le t_i < \overline{T}) = F_j(\beta_j^p)\int_0^{T^P} q_j(t)[1-k_it]dt + [F_j(\beta_j^f)-F_j(\beta_j^p)]\int_{T^p}^{T^f} s_j(t)[1-k_i(t)]dt$$
$$+\int_{T^f}^{t_i} f_j(\tau_j(t,0))\tau_j'(t,0)[1-k_i(t)]dt - \int_{T^p}^{t_i}(\tau_j(t,1))\tau_j'(t,1)[w_i-k_it]dt$$
$$+[F_j(\tau_j(t_i,1))-F_j(\tau_j(t_i,0))]\max\{-a_it-k_it, w_i-k_it\} \tag{A. 9}$$

Next, Taking the derivative of (A. 9) twice with respect to t_i, we find that a resolved type's expected utility function will be concave if

$$\frac{df_j(\tau_j(t_i,0))\tau_i'(t_i,0)}{dt}[1-\overline{w}_i] - k_i[f_j(\tau_j(t_i,1))\tau_j'(t_i,1) - f_j(\tau_j(t_i,0))\tau_i'(t_i,0)] < 0$$

To find the value of (13), we can rearrange country j's strategy in equation (12) to find that

$$f_j(\tau_j(t,0))\tau_i'(t,0)[1-\overline{w}_i^t] = k_i[F_j(\tau_j(t,1)) - F_j(\tau_j(t,0))]$$

Taking the derivative with respect to t have

$$\frac{df_j(\tau_j(t,0))\tau_i'(t,0)}{dt}[1-\overline{w}_i^t] - \frac{\overline{w}_i^t}{dt}f_j(\tau_j(t,0))\tau_i'(t,0) = k_i[f_j(\tau_j(t_i,1))\tau_j'(t_i,1) - f_j(\tau_j(t_i,0))\tau_j'(t_i,0)]$$

Substituting this back into the second derivative of country i's utility function we are left with

$$\frac{d\overline{w}_i^t}{dt}f_j(\tau_j(t_i,0))\tau_j'(t_i,0) < 0$$

which is negative as desired given that $\tau_j'(t_i,0)$ is increasing and that $\frac{d\overline{w}_i^t}{dt}$ is decreasing in t per Lemma 4.

Finally, we examine the second order condition for a type that concedes. Taking the derivative of (A. 9) twice with respect to t_i, we find that an unresolved type's expected utility function will be concave if

$$\frac{f_j(\tau_j(t_i,0))\tau_j'(t_i,0)}{dt}[1+a_it_i] + f_j(\tau_j(t_i,0))\tau_j'(t_i,0)[k_i+2a_i]$$
$$-\frac{f_j(\tau_j(t_i,1))\tau_j'(t_i,1)}{dt}[\underline{w}_i + a_it_i] - f_j(\tau_j(t_i,1))\tau_j'(t_i,1)[k_i+2a_i]$$

Once again, we cannot proceed without additional information relating to the derivative of the τ terms. We know from the first order condition of a conceding type of country i that it is possible to express the hazard rate for concession as

$$\frac{f_j(\tau_j(t,0))\tau_j'(t,0)}{F_j(\tau_j(t,1)) - F_j(\tau_j(t,0))} = \frac{f_j(\tau_j(t,1))\tau_j'(t,1)}{F_j(\tau_j(t,1)) - F_j(\tau_j(t,0))}\frac{[\underline{w}_i^t + a_it_i]}{1+a_it} + \frac{a_i+k_i}{1+a_it}$$

The expression in (13) is derived after rearranging and substituting in (12) into this expression. However, to derive more information regarding the τ terms, we instead take the derive of this expression with respect to t to find that

$$\frac{f_j(\tau_j(t,0))\tau_j'(t,0)}{dt}[1+a_it] = -f_j(\tau_j(t,0))\tau_j'(t,0)[k_i+2a_i] + \frac{f_j(\tau_j(t,1))\tau_j'(t,1)}{dt}[\underline{w}_i^t + a_it)] \\ +f_j(\tau_j(t,1))\tau_i'(t,1)\left[\frac{d\underline{w}_i^t}{dt} + k_i + 2a_i\right]$$

Substituting this back into the second order condition we are left with

$$f_j(\tau_i(t_i,1))\tau_i'(t_i,1)\frac{d\underline{w}_i^t}{dt} < 0$$

which is negative as desired given that $\tau_j'(t_i,1)$ is decreasing and that $\underline{w}_i$ is increasing in t per Lemma 4. ■

A.3.5 Restatement of Proposition 3

Proposition A. 2 *Let $T^2 = \min\{T^f, \overline{T}\}$. If there exists a $T^p < \overline{T}$. During $[T^p, T^2]$, country i concedes by playing $\tau_i(\cdot, 0)$ as given by*

$$\frac{f_i(\tau_i(t,0))\tau_i'(t,0)}{1-F_i(\tau_i(t,0))} = \frac{a_j+k_j}{1+a_jt} \tag{A. 10}$$

If $T^2 = T^f$, then types $w_j \in [\beta_j^p, \beta_j^f]$ concede by playing

$$\frac{[F_j(\beta_j^f) - F_j(\beta_j^p)]s_j(t)}{F_j(\tau_j(t,1)) - [F_j(\beta_j^f) - F_j(\beta_j^p)]S_j(t) - F_j(\beta_j^p)} = \frac{a_i+k_i}{1+a_it} \tag{A. 11}$$

$$+\frac{f_j(\tau_j(t,1))\tau_j'(t,1)}{F_j(\tau_j(t,1)) - [F_j(\beta_j^f) - F_j(\beta_j^p)]S_j(t) - F_j(\beta_j^p)} \times \frac{\underline{w}_i^t + a_it}{1+a_it} \tag{A. 12}$$

on the interval $[T^p, T^2]$. If $T^2 = T^f$, then conceding types of country j may choose to play (A. 12) on the interval $[T^p, T^2)$ and play

$$s_j(\overline{T}) = \lim_{t\to\overline{T}^-} 1 - S_j(t) \tag{A. 13}$$

Resolved types of country j play

$$\sigma_j(w_j|1) = [1-w_j]\left[\frac{1}{k_j} + \frac{1}{a_j}\right] - \frac{1}{a_j} \tag{A. 14}$$

Each country's posterior beliefs posterior beliefs during this period are given by

$$g_i(w_j|t) = \begin{cases} \frac{f_j(w_j)[1-S_j(t)]}{F_j(\tau_j(t_i,1))-[F_j(\beta_j^f)-F_j(\beta_j^p)]S_j(t)-F_j(\beta_j^p)} & \text{if } w_j \in [\beta_j^p, \beta_j^f] \\ \frac{f_j(w_j)}{F_j(\tau_j(t_i,1))-[F_j(\beta_j^f)-F_j(\beta_j^p)]S_j(t)-F_j(\beta_j^p)} & \text{if } w_j \in [\beta_j^f, \overline{w}_j^t] \\ 0 & \text{otherwise} \end{cases} \tag{A. 15}$$

1

$$g_j(w_i|t) = \begin{cases} \frac{f_i(w_i)}{1-F_i(\tau_i(t,0))} & \text{if } w_i \in [\underline{w}_i^t, \overline{w}_i] \\ 0 & \text{otherwise} \end{cases} \tag{A. 16}$$

A.3.6 Proof of Proposition 3

Using Lemma 3, we can rewrite the expected utility function for an unresolved type of country i who concedes during the first screening phase as

$$\begin{aligned} U_i(t_i, 0; w_i|T^p \leq t_i \leq T^f) = F_j(\beta_j^p) \int_0^{T^p} q_j(t)[1-k_i t]dt \\ +[F_j(\beta_j^f) - F_j(\beta_j^p)] \int_{T^p}^{t_i} s_j(t)[1-k_i t]dt - \int_{T^p}^{t_i} f_j(\tau_j(t,1))\tau_j'(t,1)[w_i - k_i t]dt \\ -[F_j(\tau_j(t_i,1)) - (F_j(\beta_j^f) - F_j(\beta_j^p))S_j(t_i) - F_j(\beta_j^p)][a_i t_i + k_i t_i] \end{aligned} \tag{A. 17}$$

Similarly the utility for a type of country j who exits during the first screening phase is given by

$$\begin{aligned} U_j(t_j, \theta_j; w_j|T^p \leq t_i \leq T^f) = F_i(\beta_i^p) \int_0^{T^p} q_i(t)[1-k_j]dt \\ + \int_{T^p}^{t_j} f_i(\tau_i(t,0))\tau_i'(t,0)[1-k_j t]dt + [1-F_i(\tau(t_j,0))] \max\{-a_j t_j - k_j t_j, w_j - k_j t_j\} \end{aligned} \tag{A. 18}$$

Once again, conceding types of country j have no profitable deviation on account of their being indifferent and playing a mixed strategy. Similarly types of country j going to war have no incentive to deviate - their strategy is derived from a first-order condition that was already shown to produces a maximum in the proof of Proposition 2. The proof that that the first-order condition for a conceding type of country i produces a maximum requires showing that the utility function in (A. 17) is concave in t. The proof that this property is satisfied is identical to the proof showing that the utility for a conceding type is concave in the second screening phase and follows immediately from the proof of proposition 5.

A.3.7 Proof of Proposition 4

Using Lemma 3, we can restate type $\overline{w}_i$'s expected utility for going to war during the first screening phase as follows

$$
\begin{aligned}
U_i(t_i, 1; \overline{w}_i | T_p \leq t_i \leq T^f) = F_j(\beta_j^p) \int_0^{T^p} q_j(t)[1 - k_i t]dt \\
+[F_j(\beta_j^f) - F_j(\beta_j^p)] \int_{T^p}^{t_i} s_j(t)[1 - k_i t]dt - \int_{T^p}^{t_i} f_j(\tau_j(t,1))\tau_j'(t,1)[\overline{w}_i - k_i t]dt \\
+[F_j(\tau_j(t_i,1)) - (F_j(\beta_j^f) - F_j(\beta_j^p))S_j(t_i) - F_j(\beta_j^p)][\overline{w}_i - k_i t_i]
\end{aligned}
\tag{A. 19}
$$

To show that the most resolved type of country i has a maximum when going to war at time T^f, it is necessary to show that its utility is concave. The derivative of country i's utility function as given by equation (A. 19) with respect to t_i shows that country i will go to war when

$$
\frac{F_j(\beta_j^f) - F_j(\beta_j^p)s_j(t)}{F_j(\tau_j(t_i,1)) - (F_j(\beta_j^f) - F_j(\beta_j^p))S_j(t_i) - F_j(\beta_j^p)} = \frac{k_i}{1 - \overline{w}_i}
$$

Note that this first order condition is identical to that resulting from equation (A. 9). However, the hazard rate characterizing Country j's concession rate is given by (A. 12) instead of simply being (12). However, note that by taking the derivative of (A. 9) for a conceding country, it is possible to express the hazard rate in (12) exactly as (A. 12). It therefore follows that the escalating type of country i's utility must be concave since it is so in the the proof in Proposition 3.■

A.3.8 Restatement of Proposition 6

Proposition A. 3 *Strategic behaviour ends at $\overline{T}$, which can arrive during any phase. Countries' choice of exit strategy and their behavior at the horizon date is determined by the following:*

(i) All types exit by the horizon date: There exists an equilibrium where types $w_i \in [\underline{w}_i, \beta_i]$ concede and types $w_i \in (\beta_i, \overline{w}_i]$ go to war where $\beta_i = -a_i\overline{T}$. Any type still participating in the crisis at $\overline{T}$ goes to war at that time.

(ii) One country exits by the horizon date: If $\overline{T} < T^f$, then there exists an equilibrium where types $w_i \in [\underline{w}_i, \beta_i]$ concede and types $w_i \in (\beta_i, \overline{w}_i]$ go to war where

$$
\beta_i = \frac{-a_i(\overline{T}) + \mu F_j(\beta)}{1 - F_j(\beta_j)} \tag{A. 20}
$$

and $\mu = q_j(\overline{T})$ if $\overline{T} < T^p$ and $\mu = s_j(\overline{T})$ otherwise. In turn, types $w_j \in [\underline{w}_j, \beta_j]$ concede and types $w_j \in (\beta_j, \overline{w}_j]$ go to war where $\beta_j = -a_j\overline{T}$.

(iii) Some types remain in forever: If $K_i < a_i\overline{T}$ for both $i = 1, 2$, then there exists an equilibrium

where types $w_i \in [\underline{w}_i, \beta_i]$ concede for β_i as given by

$$\frac{F_j(\overline{w}_j^{\overline{T}}) - F_j(-\overline{K}_j)}{F_j(\overline{w}_j^{\overline{T}}) - F_j(\beta_j)}\beta_i - \frac{F_j(-\overline{K}_j) - F_j(\beta_j)}{F_j(\overline{w}_j^{\overline{T}}) - F_j(\beta_j)}K_i = -a_i\overline{T} \tag{A. 21}$$

Types $w_i \in (\beta_i, -K_i]$ remain in the crisis forever and types $w_i \in (-K_i, \overline{w}_i]$ go to war. Any type from the latter set still participating in the crisis at $\overline{T}$ go to war at that time.

A.3.9 Proof of Proposition 6

Parts (i) and (iii) of the proof follow directly from the arguments in the main text. For part (ii), note that type β_i is the type that is indifferent between conceding and paying $a_i\overline{T}$ audience costs and remaining in the war of attrition and obtaining a concession with probability $q_j(\overline{T})F_j(\beta_j)$ and going to war with probability $1 - F_j(\beta_j)$. It follows that any type with a lower wartime payoff than β_i must strictly prefer to concede and any type stronger than β_i will prefer to remain in the crisis and risk going to war. Type β_j is the type indifferent between paying audience costs and going to war. It follows that any type with a lower wartime payoff prefers to concede and any type with a larger wartime payoff prefers to go to war.

Note that this requires that remaining resolved types of Country i play a strategy that has them exit at a time $t_i > \overline{T}$. In this case, country j has no incentive to deviate since the only types of country i remaining at $\overline{T}$ are resolved types who wish to fight, implying that delay can no longer lead to a concession and will increase the amount of sunk costs paid. Similarly, resolved types of Country i have no interest in going to war at an earlier time as per Propositions 2 and 4 since $\overline{T} < T^f$.

Moreover, observe that a stalemate is not possible when $q_j(\overline{T}) > \epsilon$ or $s_j(\overline{T})\epsilon$ for an arbitrarily small $\epsilon > 0$. This is because resolved types of Country i would not go to war at time $\overline{T}$, since they would prefer to wait until country j finished conceding. However, this would then require that any type $w_i \in [-\overline{K}_i, \overline{w}_i]$ go to war at a time $t > \overline{T}$, a violation of Lemma 1. ■ ■

Note that a stalemate is not possible if one type has ma

B Ruling out Alternative Equilibria

In this section I rule out (i) an equilibrium in which the war of attrition ends with probability 1 at $t = 0$ and (ii) any equilibrium which has types exit after time $t = 0$ and which does not feature the three phases described in the main text in the order that they are described. That is, I show that the equilibrium must begin with a peaceful phase, before transitioning into the first screening phase, and only then to the second screening phase.[5] The first proposition demonstrates that the war of attrition must proceed past $t = 0$ with positive probability

[5] As mentioned in the main text there is an exception in that the first screening phase can be skipped when resoved types of both countries want to go to war at the same time at the end of the peaceful phase.

Proposition B. 1 *The war of attrition must feature a positive probability of concession and war after* $t = 0$*.*

The proof of this result follows directly from the way simultaneous exit is determined in the expected utility function in (A. 1). Since the outcome of the model (i.e. war or concession) is adjudicated by a simple coin toss, there are strong pressures against going to war or conceding when one's rival is likely to concede with positive probability. This prohibits all countries from exiting immediately at $t = 0$, since there is an incentive to delay if one's rival is going to concede at $t = 0$ with positive probability.

The following proposition demonstrates the second result.

Proposition B. 2 *Any equilibrium that does not have the war of attrition end at* $t = 0$*, must have countries play strategies according to Propositions 1-5.*

The proof of this proposition relies on the fact that Lemmas 2, 3, and 4 still apply to any interval of time in which neither country has types go to war, one country has types go to war, or both countries have types go to war. It then shows that continuity and monotonicity and properties perscribed by these lemma are incongruent with countries going to war prior to when is stipulated by Propositions 1 through 5.

Proof of Proposition B. 1

It cannot be the case that resolved types of both countries cannot go to war at time $t = 0$. This is because unresolved types $w_i < 0$ for $(i = 1, 2)$ would respond by trying to concede at $t = 0$ implying that resolved types could increase their expected utility from $F_j(0)\frac{1+w_i}{2} + (1 - F_j(0))w_i$ to $F_j(0) + (1 - F_j(0))w_i$ by delaying concession by an arbitrarily small ϵ. Similarly, it cannot be the case that both countries concede at $t = 0$. As one country could strictly increase its utility from $\frac{1}{2}F_j(0)$ to $F_j(0)$ by delaying concession by an arbitrarily small ϵ. ■

Proof of Proposition B. 2

Any alternative equilibria must contain intervals where neither country has any types that fight, intervals where only one country has types that fight, or intervals where both countries have types that fight. Lemmas 2, 3, and 4 respectively would still apply to such intervals in any alternative equilibrium.

The requirement in Lemma 4 that $\sigma_i(\cdot|1)$ be continuous and strictly decreasing rules out equilibria that begin with an interval in which both countries go to war too early. To see why simply note that this requirement implies that (13) must be the hazard rate that determines when countries go to war in any such alternative equilibrium. Observing $\sigma_i(\cdot|1)$, we note that the denomenator is always negative and that the numerator will only be negative as required if

$$k_i[\overline{w}_i^t + a_i t] > a_i[1 - w_i]$$

Rearranging, this requires that

$$\frac{k_i}{1-\overline{w}_i^t} > \frac{a_i + k_i}{1 + a_i t}$$

We know from the discussion of the peaceful phase, that this will not hold for a single type until T^p. It follows that the game cannot begin with an interval in which both sides concede as both countries would be unable to play a strategy $\sigma_i(\cdot|1)$ that is continuous and strictly decreasing.

Therefore any alternative equilibrium must begin with an interval where at least one country does not have any escalating types and the other does. Without loss of generality, let country j be the country to have the type(s) escalating in such an interval. From the discussion of the first screening phase, we know that any such interval must have conceding types of country i play a strategy that keeps conceding types of country j indifferent and must therefore be given by (A. 10). However, from the proof of Proposition 2, we know that when country i concedes using this hazard rate that no type of country j will choose to escalate prior to T^p. It follows that the equilibrium where countries play according to the strategies described in Propositions 1-5 is the only type of equilibrium.■

C Comparison to Fearon (1994)

In the main text, I describe three differences between my model and that in Fearon (1994): (i) the addition of sunk costs for delay, (ii) the fact that states can have a positive payoff for fighting, and (iii) relaxing the assumption that war occurs in finite time. In this section, I provide a concise discussion of the differences and how their impact.

C.1 Difference 1: The Introduction of Sunk Costs

As discussed in the introduction in the main text, my model includes sunk costs whereas Fearon (1994) does not. This is a necessary condition for resolved types to be screened so that higher resolve types exit the crisis and go to war earlier. Intuitively, by making delay costly, the sunk costs cause those states with more appealing outside options to abandon crisis negotiations earlier in favor of going to war. By contrast, if there are no sunk costs then delay is cost free for any type who wants to go to war. This would cause resolved types to wait until they were certain that their rival would not concede before choosing to fight as in Fearon (1994).

C.2 Difference 2: Allowing for Types with Positive Payoffs to Fighting:

Fearon (1994) assumed that all types of both countries had negative payoffs for fighting . Formally, $\bar{w}_i \leq 0$. Fearon made this assumption to show that audience costs could lock states into conflict even when severe assumptions against war were imposed. Though I relax this assumption in the main text, and allow for $\overline{w}_i > 0$, screening of resolved types by sunk costs is still possible even if

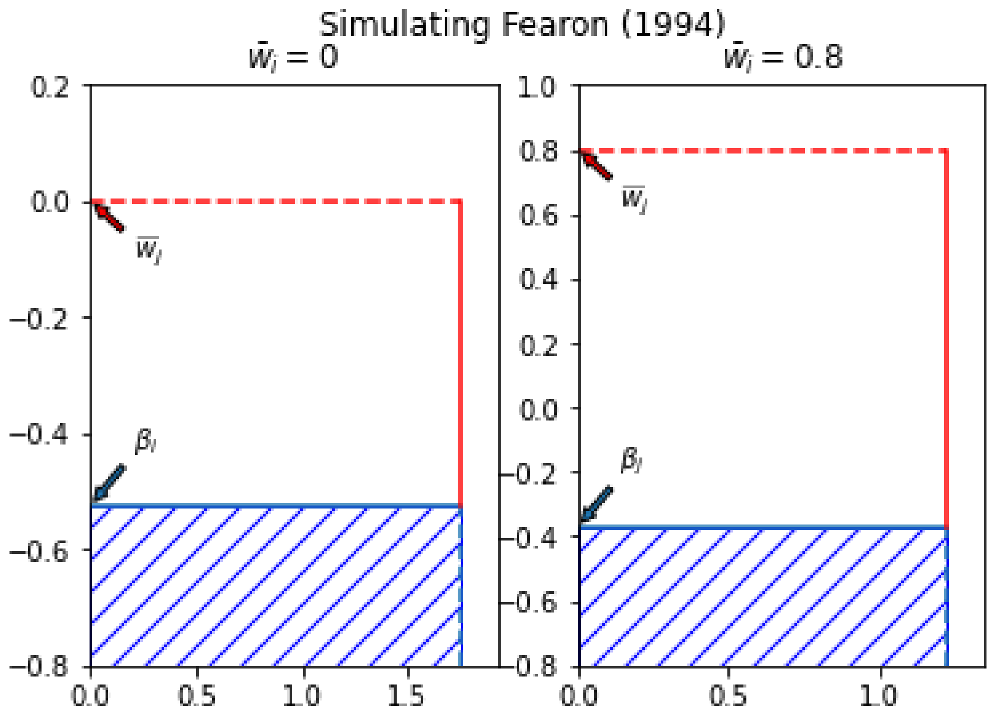

Figure C. 1: **Simulating Fearon (1994):** Both panels present strategies adopted by the two countries when there are no sunk costs for delay. In the left panel $\overline{w}_i = 0$, as in Fearon (1994). In the right panel $\overline{w}_i = 0.8$. So long as delay imposes no sunk costs, the equilibrium will only ever consist of a peaceful phase. In both panels $\underline{w}_i = -0.8$, $a_i = 0.3$ and $F_i(\cdot)$ is a uniform distribution.

$\overline{w}_i = 0$. However, in general, the lower is $\overline{w}_i$, the higher sunk costs will need to be for screening of resolved types to occur.

To demonstrate this point consider the following series of numerical simulations of the model in the main text. To simplify matters, I will assume that both countries are symmetric such that if sunk costs do screen resolved types, the peaceful phase will proceed directly into the second screening phase. In all the examples that follow, I assume that $\underline{w}_i = -0.8$, $a_i = 0.3$ and that $F_i(\cdot)$ is a uniform distribution. Across example, I will vary $\overline{w}_i$ and k_i to illustrate the degree of sunk costs required to induce screening behavior.

First, consider this first example in the left panel of the Figure (C. 1) designed to replicate Fearon (1994) such that $\overline{w}_i = 0$ and $k_i = 0$. Following the logic described above, without sunk costs resolved types are never screened and the game ends in a peaceful phase regardless of of the value of $\overline{w}_i$. This is illustrated by the figure in the right-panel illustrating the equilibrium that occurs with $\overline{w}_i = 0.8$ and $k_i = 0$.

Second, figure (C. 2) screening is possible once we introduce sunk costs, even if $\overline{w}_i = 0$. In the left panel, k_i has been set to $k_i = 1.2$ and has led to a short screening phase. This value of k_i, four times the value of a_i, is the minimum k_i that can cause screening for these parameter values. when k_i is increased in increments of 0.1. The right panel simulates the model when k_i is increased

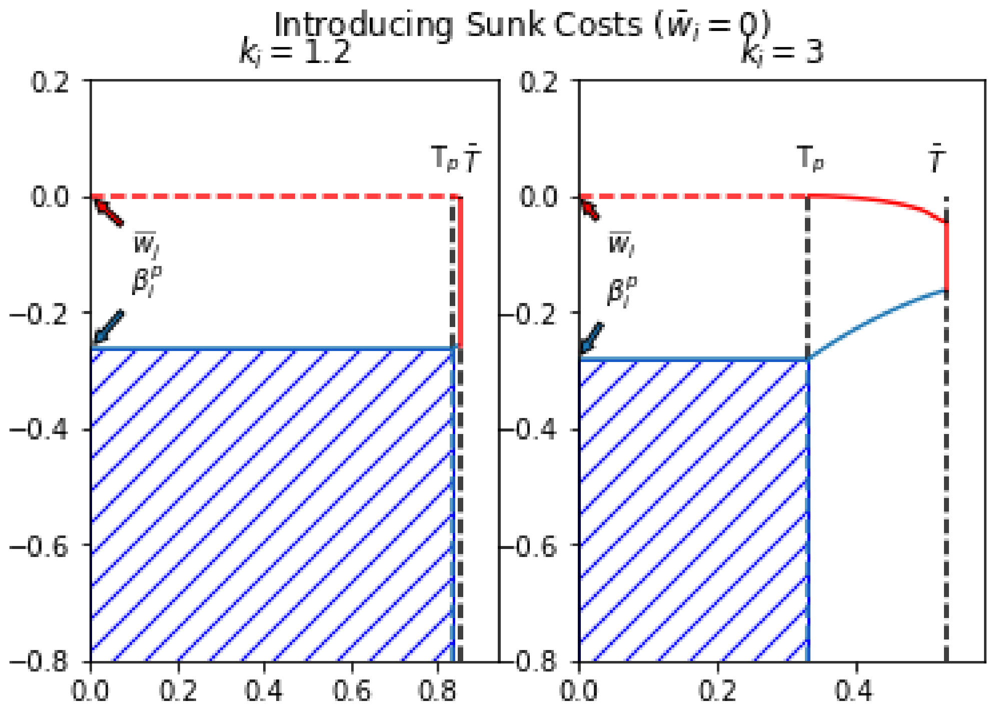

Figure C. 2: **Introducing Sunk Costs ($\overline{w}_i = 0$):** Both panels present strategies adopted by the two countries when $\overline{w}_i = 0$. In the left panel k_i has been set to 1.2, four times the value of audience costs, and a short screening phase can be achieved. The right panel has had k_i set to 3, ten times the value of audience costs and has a relatively longer screening phase. In both panels $\underline{w}_i = -0.8$, $a_i = 0.3$ and $F_i(\cdot)$ is a uniform distribution.

to a value of 3, ten times the value of audience costs. However, in general screening of resolved states is more easily as the upper bound of each country's possible resolve increases. Figure (C. 3) demonstrates this. In it's right panel, the figure demonstrates that screening is possible when audience costs and sunk costs both have a value of 0.3 for $\overline{w}_i = 0.4$, the minimum value of $\overline{w}_i$ for which this is possible (in increments of 0.1). The left panel demonstrates that a short screening phase is possible when $k_i = 0.5$ for the interim value of $\overline{w}_i = 0.2$. In the latter case this is the minimum value of sunk costs (in incremenets of 0.1) for which screening of resolved types can be achieved for this particular valye of $\overline{w}_i$.

C.3 Difference 3: Crises Must End in Finite Time:

In Fearon (1994), the requirement that crises end in finite time was necessary to make sense of the results. As in the model in the main text, in Fearon (1994) there is an endogenous horizon date after which no type concedes. Because Fearon's model did not include sunk costs, the remaining resolved types of the two countries, who are all presumed to have negative payoffs to fighting, have no incentive to go to war and would prefer to delay in perpetuity. To circumvent this problem, Fearon restricts attention to cases where the remaining types past the horizon date play a strategy

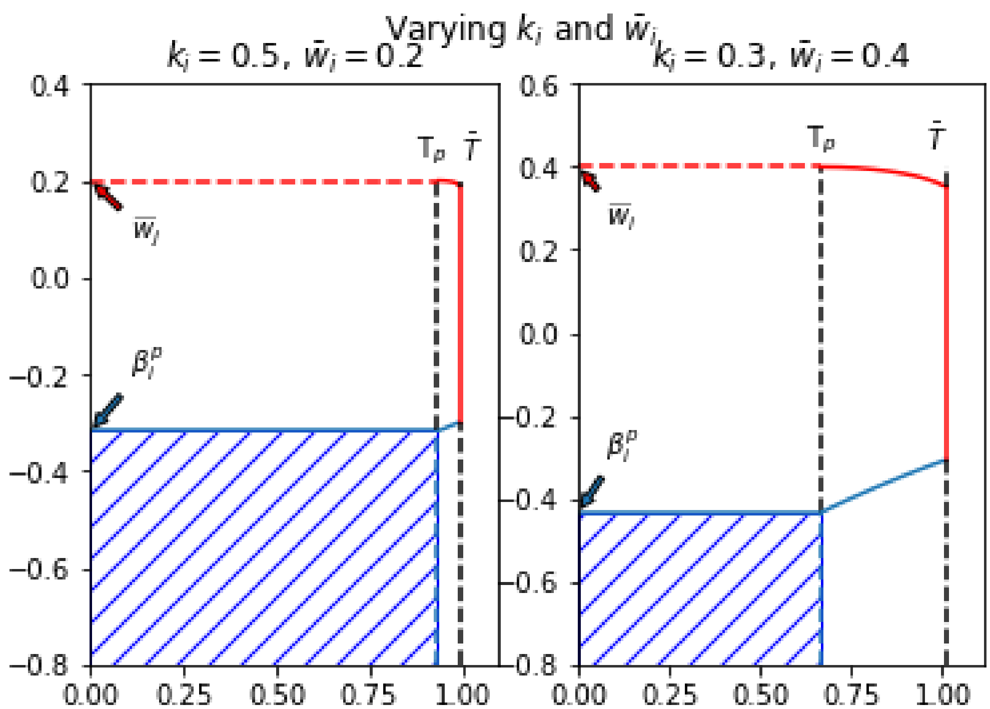

Figure C. 3: **Varying $\overline{w}_i$:** Thios figure demonstrates that when w_i is allowed to increase, screening of resolved types by sunk costs can occur with lower values of k_i. The right-panel has $\overline{w}_i = 0.4$, the minimum value of $\overline{w}_i$ (in incremenets of 0.1) for which audience costs and sunk costs can both have a value of 0.3 and have screening occur. The left panel demonstrates that for the interim value of $\overline{w}_i = 0.2$, a short screening phase is possible with sunk costs having a value of 0.5. In both panels $\underline{w}_i = -0.8$, $a_i = 0.3$ and $F_i(\cdot)$ is a uniform distribution.

where they both go to war *at some point* past the horizon date - absent any sunk costs penalizing delay, war could occur at any time $t \in [\overline{T}, \infty)$. Even though the remaining resolved types would prefer not to fight, an equilibrium exists if at any time $t \in [\overline{T}, \infty)$ all remaining types of both countries played (fight, fight) such that neither could change the outcome by unilateral deviation. This is discussed in the section characterizing the horizon date and is the reason why a stalemate need not occur, even the necessary conditions for a stalemate (sufficiently low $\bar{K}_i$ for $(i = 1, 2)$) are met.

As mentioned in the main text, Fearon (1994) does not acknowledge the potential for a stalemate. This is because if a stalemate occured then all types would opt for a stalemate - since delay is not costly and there are no types who can credibly threaten war, no type would ever concede.[6] Thus Fearon's results are not robust to the introduction of stalemates. By contrast, in the model presented in the main text, the imposition of sunk costs (in the form of the penalty $\overline{K}$) mean that resolved types do not want to delay and that types sufficiently resolved not to concede may prefer war to stalemate.

D Modeling Extension: Introducing a Discount Factor

In the main text, I assume that states stop incurring sunk costs if the war of attrition enters a stalemate and incur a one-time penalty instead. In this section, I assume that countries pay sunk costs indefinitely and introduce a discount factor. To some readers this might allow for a more natural method with which to bound bound payoffs and allow for endogenous stalemates. I show that the results in the main text are robust to the introduction of a discount factor in that they are qualitatively similar to those in the main paper.

Let $e^{-r_i t}$ denote the discount factor incurred by a state who chooses to wait until time t with discount rate $r_i > 0$. We can rewrite the utility function in equation (A. 1) as

[6]This assumes that $\bar{K}_i = 0$ in Fearon's model, in the spirit of no costs for delay.

$$
\begin{aligned}
U_i(t_i,\theta_i,\sigma_j|w_i) = &\int_{\{w_j|t_j<t_i,\theta_j=0\}} [f_j(w)(1-k_i\sigma(w|0))e^{-r_i\sigma(w|1)}]dw \\
&+ \int_{\{w_j|t_j<t_i,\theta_j=1\}} [f_j(w)(w_i-k_i)\sigma(w|1)]e^{-r_i\sigma(w|1)}dw \\
&+\mathbb{1}_{\{t_i\neq\infty,\theta_i=0\}}\Bigg[\int_{\{w_j|t_j=t_i,\theta=0\}} \left[f(w)\left(\frac{1}{2}(1-a_it_i)-k_it_i\right)e^{-rt_i}\right]dw+ \\
\int_{\{w_j|t_j=t_i,\theta=1\}} &\left[f(w)\left(\frac{1}{2}(w_i-a_it_i)-k_it_i\right)e^{-rt_i}\right]dw - \int_{\{w_j|t_j>t_i)\}} [f_j(w)(a_it_i+k_it_i)]e^{-rt_i}dw\Bigg] \\
&+\mathbb{1}_{\{t_i\neq\infty,\theta_i=1\}}\Bigg[\int_{\{w_j|t_j=t_i,\theta=0\}} \left[f(w)\left(\frac{1}{2}(1+w_i)-k_it_i\right)e^{-rt_i}\right]dw \\
+\int_{\{w_j|(t_j=t_i,\theta=1)\vee(t_j>t_i)\}} &\left[f(w)\left(w_i-k_it_i\right)e^{-rt_i}\right]dw\Bigg] - \mathbb{1}_{\{t_i=\infty\}}\int_{\{w_j|t_j=\infty)\}}[f(w)(K_i+k_i\overline{T})e^{-r\overline{T}}]dw
\end{aligned}
\tag{D. 1}
$$

In what follows, I replicate each of the propositions in the main text. The lemmas replicate in a much more straightforward manner, and will not be restated.

D.1 The Peaceful Phase with a Discount Factor

As in the main text, the game begins with a peaceful phase during which neither country goes to war. Conceding types must play a mixed strategy $Q_i(t)$ during this phase and Lemma 2 from the main text continues to apply.

The expected utility function for a type that concedes during the peaceful phase is

$$
U_i(t_i,0;w_i|t<T^p) = F_j(\beta_j^p)\int_0^{t_i} q_j(t)[1-k_it]e^{-r_it}dt - [1-Q_j(t_i)F_j(\beta_j^p)])[a_it_i+k_it_i]e^{-r_it} \tag{D. 2}
$$

Taking the first-order condition, we can characterize the modified hazard rate as

$$
\frac{F_j(\beta_j^p)q_j(t)}{1-Q_j(t_i)F_j(\beta_j^p)} = \frac{[a_i+k_i][1-r_it]}{1+a_it} \tag{D. 3}
$$

The hazard rate reveals that the introduction of the discount factor causes the rate of concession to slow down as the war of attrition progresses. This is required to adjust for the way in which the discount factor ameliorates the penalties from the audience costs and sunk costs that accumulate with delay.

To determine when the most resolved type of either country will go to war, we can rewrite equation (A. 8), the expected utility function for a type going to war during the peaceful phase, as

$$
U_i(t_i,1;\overline{w}_i|t\leq T^p) = F_j(\beta_j^p)\int_0^{t_i} q_j(t)[1-k_it]e^{-r_it}dt + [1-F_j(\beta_j^p)Q_j(t_i)][\overline{w}_i-k_it_i]e^{-r_it_i} \tag{D. 4}
$$

Taking the first order condition we find that type $\overline{w}_i$'s utility will be maximized when

$$\frac{F_j(\beta_j^p)q_j(t_i)}{1 - Q_j(t_i)F_j(\beta_j^p)} = \frac{k_i[1 - r_i t_i] + r_i \overline{w}_i}{1 - \overline{w}_i} \tag{D. 5}$$

This mirrors the calculations in the main text with the addition of an $r\overline{w}_i$ term in the numerator on the right-hand side. This reflects that the discount factor makes the most resolved type more impatient and willing to exit earlier since the discount factor reduces the gains from a delayed war. Substituting equation (D. 3) into the expression, we find that the most resolved type of country i will choose to go to war when

$$\overline{w}_i = \frac{1 - \frac{k_i}{a_i + k_i}[1 + a_i t_i]}{1 + \frac{r_i[1 + a_i t_i]}{[a_i + k_i][1 - r_i t]}} \tag{D. 6}$$

This is similar to the expression in equation (4) in the main text, with the addition of the terms in the denominator on the right-hand side. These additional terms are strictly greater than 1 for an $r > 0$. Since the numerator is strictly decreasing in t, it follows that the discount factor causes the peaceful phase to end sooner.

As in the main text, the peaceful phase will end whenever there is a single type that is no longer willing to delay going to war. Defining T^p analogously to its definition in the main text, we can restate Propositions 1 and 2 as follows

Proposition D. 1 *Let* $T_1 = \min\{T^p, \overline{T}\}$

(i) Types $w_i \in [\underline{w}_i, \beta_i^p]$ *(*$i = 1, 2$*) concede during* $[0, T^1]$ *and play strategy* $Q_i(t)$ *as defined in equation (D. 3).*

(ii) No type goes to war during the interval $[0, T^1)$.

(iii) Countries posterior beliefs during this period are given by equation (A. 7).

Proof: To replicate the proof of Proposition 1, we need to show that the utility function in equation (D. 4) is concave in t. Taking the second derivative, we find that this will be true if

$$F_j(\beta_j^p)\frac{dq_j(t_i)}{dt}[1 - \overline{w}_i] - k_i q_j(t_i) + r_i q_j(t_i)[\overline{w}_i - k_i t_i] + r_i k_i[1 - F_j(\beta_j^p)Q_j(t_i)] < 0$$

As in the main text, to find the value of $F_j(\beta_j^p)$ we must rearrange the hazard rate in equation (D. 3) to find that

$$q_j(t)[1 + a_i t_i] = [a_i + k_i][1 - rt][1 - F(\beta^p)Q_j(t)]$$

so that taking the derivative with respect to t, we find that

$$F_j(\beta_j^p)\frac{dq_j(t)}{dt} = \frac{-r_i[a_i + k_i][1 - F(\beta^p)Q_j(t)] - q_j(t)[2a_i + k_i] + r_i q_j(t)[a_i t + k_i t]}{1 + a_i t}$$

Which we can substitute back into the second order condition

$$[1-\overline{w}_i]\left[\frac{-r_i[a_i+k_i][1-F_j(\beta_j^p)Q_j(t_i)]-q_j(t_i)[2a_i+k_i]+r_iq_j(t_i)[a_it_i+k_it_i]}{1+a_it_i}\right]$$
$$-k_iq_j(t_i)+r_iq_j(t_i)[\overline{w}_i-k_it_i]+r_ik_i[1-F_j(\beta_j^p)Q_j(t_i)]<0$$

Rearranging and substituting for the hazard rate from equation (D. 3), we find that

$$[1-\overline{w}_i]\left[-r_i[a_i+k_i]-\frac{[a_i+k_i]^2[1-r_it_i]^2}{1+a_it_i}-a_i\frac{[a_i+k_i][1-r_it_i]}{1+a_it_i}\right]$$
$$-k_i[a_i+k_i][1-r_it_i]+r_i[\overline{w}_i-k_it_i][1-r_it_i][a_i+k_i]+r_ik_i[1+a_it_i]<0$$

Rearranging and substituing for $\overline{w}_i$ from equation (D. 6), we are left with

$$-k_i[a_i+k_i][1-r_it_i]-r_ik_it_i[1-r_it_i][a_i+k_i]-r_i^2k_it_i[1+a_it_i]$$
$$-k_ir_i[1+a_it_i]-k_i[a_i+k_i][1-r_it_i]-k_ir_i[1-r_it_i][1+a_it_i]$$
$$-a_ik_i[1-r_it_i]-r_ia_i-\frac{r_i^2a_i[1+a_it_i]}{[a_i+k_i][1-r_it_i]}<0$$

which, given that $1-r_it_i$ must be positive to satisfy Lemma 2, is negative as desired. ■

D.2 The First Screening Phase with a Discount Factor

As in the main text, the peaceful phase is followed by a first screening phase where the resolved types of one country are screened by sunk costs while the other country's resolved types continue to delay going to war. Without loss of generality let country j be the country whose types go to war during the first screening phase. As a result, the unresolved types of country i are screened by the threat of war, while the unresolved type of country j continue to play a mixed strategy.

As in the main text, the mixed strategy played by country i must continue to concede at the same rate as in the peaceful phase, but must now does so as a monotonically increasing pure strategy

$$\frac{f_i(\tau(t,0))\tau_i'(t,0)}{1-F_i(\tau_i(t,0))}=\frac{[a_j+k_j][1-r_jt]}{1+a_jt_j} \tag{D. 7}$$

That country i's rate of concession does not change, implies that resolved types of country j face an identical trade-off as to when to exit as they did in the peaceful phase, so that (D. 6) continues to determine when resolved types of country j escalate in the first screening phase. Taking the derivative of (D. 6) with respect to t_j we find that the inverse of the strategy function for resolved types of country j is given by

$$\frac{d\overline{w}_j^t}{dt} = \frac{-\frac{k_j a_j}{a_j+k_j}\left[1+\frac{r_j[1+a_jt_j]}{[a_j+k_j][1-r_jt_j]}\right] - \left[1-\frac{k_j}{a_j+k_j}[1+a_jt_j]\right] \times \frac{r_ja_j[a_j+k_j[1-r_jt_j]+r_j^2[a_j+k_j][1+a_jt_j]}{[a_j+k_j]^2[1-r_jt_j]^2}}{\left[1+\frac{r_j[1+a_jt_j]}{[a_j+k_j][1-r_jt_j]}\right]^2} \tag{D. 8}$$

which is still negative and represents a faster rate of decrease than in equation (A. 14), a result of the decrease in expected utility from delay that is attributable to the discount factor.

The utility function for an unresolved type of country i conceding during the first screening phase is given by

$$\begin{aligned} U_i(t_i,0;w_i|T^p \leq t_i \leq T^f) = F_j(\beta_j^p)\int_0^{T^p} q_j(t)[1-k_it]e^{-rt}dt \\ +[F_j(\beta_j^f)-F_j(\beta_j^p)]\int_{T^p}^{t_i} s_j(t)[1-k_it]e^{-rt}dt - \int_{T^p}^{t_i} f_j(\tau_j(t,1))\tau_j'(t,1)[w_i-k_it]e^{-rt}dt \\ -[F_j(\tau_j(t_i,1))-(F_j(\beta_j^f)-F_j(\beta_j^p))S_j(t)-F_j(\beta_j^p)][a_it_i+k_it_i]e^{-rt_i} \end{aligned} \tag{D. 9}$$

Taking the first order condition, we find that the hazard rate for concessions for country j is given by

$$\begin{aligned} \frac{[F_j(\beta_j^f)-F_j(\beta_j^p)]s_j(t)}{F_j(\tau_j(t,1))-(F_j(\beta_j^f)-F_j(\beta_j^p))S_j(t)-F_j(\beta_j^p)} = \\ \frac{f_j(\tau_j(t,1))\tau_j'(t,1)}{F_j(\tau_j(t,1))-(F_j(\beta_j^f)-F_j(\beta_j^p))S_j(t)-F_j(\beta_j^p)} \times \frac{\underline{w}_i^t+a_it_i}{1+a_it_i} + \frac{[a_i+k_i][1-r_it_i]}{1+a_it_i} \end{aligned} \tag{D. 10}$$

Finally, we can rewrite the utility function for the most resolved type of country i as follows

$$\begin{aligned} U_i(t_i,1;\overline{w}_i|T_p \leq t_i \leq T^f) = F_j(\beta_j^p)\int_0^{T^p} q_j(t)[1-k_it]e^{-rt}dt \\ +[F_j(\beta_j^f)-F_j(\beta_j^p)]\int_{T^p}^{t_i} s_j(t)[1-k_it]e^{-rt}dt - \int_{T^p}^{t_i} f_j(\tau_j(t,1))\tau_j'(t,1)[\overline{w}_i-k_it]e^{-rt}dt \\ +[F_j(\tau_j(t_i,1))-(F_j(\beta_j^f)-F_j(\beta_j^p))S_j(t)-F_j(\beta_j^p)][\overline{w}_i-k_it_i]e^{-rt_i} \end{aligned} \tag{D. 11}$$

Taking the first order condition, we find that the most resolved type of country i will go to war when country i concedes at the same rate as the right-hand side of equation (D. 5). Substituting for the left-hand side hazard rate from (D. 10), we find that the most resolved type of country i will go to war when

$$\overline{w}_i = \frac{1-\frac{k_i'[1-r_it_i][1+a_it_i]}{\frac{f_j(\tau_j(t_i,1))\tau_j'(t_i,1)}{F_j(\tau_j(t_i,1))-(F_j(\beta_j^f)-F_j(\beta_j^p))S_j(t_i)-F_j(\beta_j^p)}[\underline{w}_i+a_it_i]+[a_i+k_i][1-r_it_i]}}{1+\frac{r_i\overline{w}_i[1+a_it_i)]}{\frac{f_j(\tau_j(t_i,1))\tau_j'(t_i,1)}{F_j(\tau_j(t_i,1))-(F_j(\beta_j^f)-F_j(\beta_j^p))S_j(t_i)-F_j(\beta_j^p)}[\underline{w}_i+a_it_i]+[a_i+k_i][1-r_it_i]}} \tag{D. 12}$$

We can therefore restate Proposition 3 and 4 as

Proposition D. 2 *Let* $T^2 = \min\{T^f, \overline{T}\}$. *If there exists a* $T^p < \overline{T}$, *then during* $[T^p, T^2]$, *the following must hold*

(i) Types w_i *who concede play strategy* $\tau_j(\cdot, 0)$ *as defined in equation (D. 7).*

(ii) Types w_j *who concede must form a connected interval and play strategy* $S_j(t)$ *as given by equation (D. 10).*

(iii) Types w_j *who go to war play* $\tau_j(\cdot, 1)$ *as defined in equation (D. 8).*

(iv) Country i *does not go to war during* $[T^p, T^2)$.

(v) County i*'s posterior beliefs posterior beliefs during this period are given by (A. 15) and beliefs for country* j *are given by (A. 16).*

where T^f is the analogue to its definition in the main text. The proof of this claim is identical to that of Proposition 3 and 4 in the main text and is therefore omitted.

D.3 The Second Screening Phase with a Discount Factor

As in the main text, the final phase of the war of attrition sees the resolved types of both countries screened by sunk costs and the unresolved types of both countries screened by the threat of war. With the introduction of a discount factor we can we rewrite the utility function for a type that exits during the second screening phase as

$$\begin{aligned} U(t_i, \omega; w_i | T^f \leq t_i < \overline{T}) = F_j(\beta^p) \int_0^{T^P} q_j(\ell)[1 - k_i t] e^{-r_i t} dt \\ +[F_j(\beta^f) - F_j(\beta^p)] \int_{T^p}^{T^f} s(t)[1 - k_i t] e^{-r_i t} dt + \int_{T^f}^{t_i} f_j(\tau_j(t, 0)) \tau_j'(t, 0)[1 - k_i t] e^{-r_i t} dt \\ - \int_{T^p}^{t_i} f(\tau_j(t, 1)) \tau_j'(t, 1)[w_i - k_i t] e^{-r_I t} dt \\ +(F_j(\tau_j(t_i, 0)) - F_j(\tau_j(t_i, 1))) \max\{w_i - k_i t_i, -a_i t_i - k_i t_i\} e^{-r_i t_i} \end{aligned} \tag{D. 13}$$

Taking the first order condition for a type that goes to war we find that the hazard rate for concessions must be given by

$$\frac{f_j(\tau_j(t, 0)) \tau_j'(t, 0)}{F_j(\tau_j(t, 0)) - F_j(\tau_j(t, 1))} = \frac{k_i[1 - r_i t] + r_i \overline{w}_i^t}{1 - \overline{w}_i^t} \tag{D. 14}$$

and that the hazard rate determining when resolved types go to war is given by

$$\frac{f(\tau_j(t, 1)) \tau_j'(t, 1)}{F_j(\tau_j(i, 1)) - F_j(\tau_j(t, 0))} = \frac{k_i[1 - r_i t_i][\overline{w}_i^t + a_i t_i] + r \overline{w}_i[1 + a_i t_i] - a_i[1 - r_i t_i][1 - \overline{w}_i^t]}{[1 - \overline{w}_i^t][\underline{w}_i^t + a_i t_i]} \tag{D. 15}$$

These closely resemble the results in the main text, though the hazard rates for both concession and escalation are accelerated. For resolved types to be willing to delay, the conceding types must compensate them for the decrease in payoff to escalation they will incur by not going to war. For conceding types, the rate of escalation must increase to account for the amelioration of the risks and costs associated with delay that result from the discount factor.

We can therefore restate proposition 5 as

Proposition D. 3 *If there exists a $T^f < \overline{T}$, then during $[T^f, \overline{T}]$ the following must hold,*

(i) Types w_i $(i = 1, 2)$ who concede play strategy $\tau_i(\cdot, 0)$ as defined in equation (D. 14).

(ii) Types w_i $(i = 1, 2)$ who go to war play strategy $\tau_i(\cdot, 1)$ as defined in equation (D. 15)

(iii) Country i's $(i = 1, 2)$ posterior beliefs during $[T^f, \overline{T})$ are given by (14).

Proof: To replicate Proposition 5, we need to show that the utility function in equation (D. 13) is concave in t_i for both a type going to war and a conceding type. Taking the second derivative we find that this will be true for a type going to war if

$$\frac{df_j(\tau_j(t_i,0))\tau_j'(t_i,0)}{dt}[1-\overline{w}_i] - f_j(\tau_j(t_i,1))\tau_j'(t_i,1))[k_i(1-r_it_i)+r\overline{w}_i]$$
$$+f_j(\tau_j(t_i,0))\tau_j'(t_i,0))[k_i(1-r_it_i)+r\overline{w}_i] + r_ik_i[F_j(\tau_j(t_i,0)) - F_j(\tau_j(t_i,1))] < 0$$

We cannot proceed without additional information regarding the $\frac{df_j(\tau_j(t_i,0))\tau_j'(t_i,0)}{dt}$ term. To solve for this term we can rearrange the hazard rate in equation (D. 14) as

$$f_j(\tau_j(t,0))\tau_j'(t,0)[1-\overline{w}_i] = k_i[1-r_it][F_j(\tau_j(t,0)) - F_j(\tau_j(t,1))]$$
$$+r_i\overline{w}_i^t[F_j(\tau_j(t,0)) - F_j(\tau_j(t,1))]$$

Taking the derivative we find that

$$\frac{df_j(\tau_j(t,0))\tau_j'(t,0)}{dt}[1-\overline{w}_i^t] = \frac{d\overline{w}_i^t}{dt}f_j(\tau_j(t,0))\tau_j'(t,0)$$
$$+r\frac{d\overline{w}_i^t}{dt}[F_j(\tau_j(t,0)) - F_j(\tau_j(t,1))] - r_ik_i[F_j(\tau_j(t,0)) - F_j(\tau_j(t,1))]$$
$$+f_j(\tau_j(t,1))\tau_j'(t,1)[k_i(1-r_it)+r_i\overline{w}_i^t] - f_j(\tau_j(t,0))[k_i(1-r_it_i)+r_i\overline{w}_i^t]$$

Substituting back into the second order condition we are left

$$\frac{d\overline{w}_i}{dt}f_j(\tau_j(t_i,0))\tau_j'(t_i,0) + r\frac{d\overline{w}_i}{dt}[F_j(\tau_j(t_i,0)) - F_j(\tau_j(t_i,1))] < 0$$

which is true given that $\overline{w}_i^t$ is strictly decreasing in t.

Similarly the second derivative for a conceding type will be given by

$$\frac{df_j(\tau_j(t_i,0))\tau_j'(t_i,0)}{dt}[1+a_it_i] - \frac{df_j(\tau_j(t_i,1))\tau_j'(t_i,1)}{dt}[\underline{w}_i+a_it_i]$$
$$+f_j(\tau_j(t_i,0))\tau_j'(t_i,0)\{a_i+[a_i+k_i][1-r_it_i]\} - f_j(\tau_j(t_i,1))\tau_j'(t_i,1)\{a_i+[a_i+k_i][1-r_it_i]\}$$
$$+r[F_j(\tau_j(t_i,1)) - F_j(\tau_j(t_i,0))][a_i+k_i] < 0$$

Note that the hazard rate that determines when resolved types go to war is first derived by taking the first order condition for a conceding type and rearranging to get

$$\frac{f_j(\tau_j(t,0))\tau_j'(t,0)}{F_j(\tau_j(t,1)) - F_j(\tau_j(t,0))} = \frac{f_j(\tau_j(t,1))\tau_j'(t,1)}{F_j(\tau_j(t,1)) - F_j(\tau_j(t,0))}\frac{\underline{w}_i^t+a_it}{1+a_it} + \frac{[a_i+k_i][1-r_it]}{1+a_it_i}$$

rather than substituting in for the left-hand side with equation (D. 14) and getting (D. 15), we rearrange to show that it is possible to express $f_j(\tau_j(t_i,1))\tau_j'(t_i,1)$ as

$$f_j(\tau_j(t,1))\tau_j'(t,1)[\underline{w}_i^t+a_it] = f_j(\tau_j(t,0))\tau_j'(t,0)[1+a_it]$$
$$-[a_i+k_i][1-r_it][F_j(\tau_j(t,1)) - F_j(\tau_j(t,0))]$$

Taking the derivative with respect to t we find that

$$\frac{f_j(\tau_j(t,1))\tau_j'(t,1)}{dt}[\underline{w}_i^t+a_it] = \frac{dtf_j(\tau_j(t,0))\tau_j'(t,0)}{dt}[1+a_it]$$
$$-f_j(\tau_j(t,1))\tau_j'(t,1)\left[\frac{d\underline{w}_i^t}{dt}+a_i+[a_i+k_i][1-r_it]\right] + f_j(\tau_j(t,0))\tau_j'(t,0)\left[a_i+[a_i+k_i][1-r_it]\right]$$
$$+r_i[a_i+k_i][F_j(\tau_j(t,1)) - F_j(\tau_j(t,0))]$$

Substituting these terms back into the second order condition, we are only left with

$$f_j(\tau_j(t_i,1))\tau_j'(t_i,1)\frac{d\underline{w}_i}{dt} < 0$$

which holds given that $\tau_j'(t_i,1)$ is strictly negative and $\frac{d\underline{w}_i}{dt}$ is strictly positive ■

D.4 The Horizon Date with a Discount Factor

As in the main text, the game ends once all unresolved types have conceded. At this point, any type of either country that intends to escalate does so as they can no longer justify delay. If a forward the valuation that a country places on paying sunk costs forever is sufficiently low, then moderately resolved types may prefer to remain in the war of attrition and incur sunk costs than pay the audience costs required to concede or the costs of fighting required for going to war.

If the war of attrition ends in a stalemate, then a type that remains in the war of attrition forever can expect to pay $\int_{\overline{T}}^{\infty} k_i' e^{-r_it}dt = \frac{k_i}{r_i}$ sunk costs. Therefore the least resolved type to escalate the crisis will be type $\alpha_i \equiv -\frac{k_i}{r_i}$ and the lest resolved type to remain in the war of attrition forever will be the type denoted β_i for which the following equation holds with equality

$$\frac{F_j(\overline{w}_j^{\overline{T}}) - F_j(\alpha_j)}{F_j(\overline{w}^t) - F_j(\beta_j)}\beta_i - \frac{F_j(\alpha_j) - F_j(\beta_j)}{F_j(\overline{w}_j^{\overline{T}}) - F_j(\beta_j)}\frac{k_i}{r} = -a_i t \tag{D. 16}$$

If the war of attrition ends with all types exiting, then the assignment of types to exit strategies is exactly as it is in the main text. We can therefore restate Proposition 6 as follows. A proof of the Proposition is omitted as it follows the arguments in the main text.

Proposition D. 4 *The game can end at any phase with the following determining countries' choice of exit strategy*

(i) All types exit: Types $w_i \in [\underline{w}_i, \beta_i)$ concede and types $w_i \in [\beta_i, \overline{w}_i]$ escalate for $\beta_i = -a_i t$.

(ii) Some types remain in forever: If $\frac{k_i}{r} < a_i t$ for both $i = 1, 2$, then there exists in equilibrium where types $w_i \in [\underline{w}_i, \beta_i)$ concede, types $w_i \in [\beta_i, \alpha_i)$ remain in the war of attrition forever, and types $w_i \in [\alpha_i, \overline{w}_i]$ escalate for $\alpha_i = -\frac{k_i}{r}$ and β_i as defined in (D. 16).

D.5 Ruling Out Alternative Equilibria with a Discount Factor

Once again, it is possible to show that any equilibrium which has types exit after time $t = 0$ must have countries exit according to the sequence of phases described above. We can therefore restate Proposition B.1 as

Proposition D. 5 *Any equilibrium that does have the war of attrition end at $t = 0$, must have countries play strategies according to Propositions C.1, C.2, and C.3.*

Proof: As in the Proof of Proposition B.1, it is the monotonicity requirements established in Lemmas 3 and 4 which rule out any alternative equilibrium. We know that equation (D. 15) must be the hazard rate that determines when counties go to war during any interval in which both countries go to war. Note that the denominator in that expression

$$[1 - \overline{w}_i][\underline{w}_i + a_i t]$$

is negative. Therefore, for $\tau_j'(t_i, 1)$ to be decreasing it must be the case that the numerator is positive. Note that the numerator

$$k_i[1 - r_i t_i][\overline{w}_i^t + a_i(t_i)] + r\overline{w}_i^t[1 + a_i t_i] - a_i[1 - r_i t_i][1 - \overline{w}_i^t]$$

will be positive so long as

$$\overline{w}_i^t > \frac{1 - \frac{k_i}{a_i + k_i}[1 + a_i t_i]}{1 + \frac{r_i[1 + a_i(t_i)]}{[a_i + k_i][1 - r_i t_i]}}$$

However, from the discussion of the peaceful phase we know that this will not be true for any type before T^p. This rules out any equilibrium that has an interval where both countries have types going to war before T^p.

The remainder of the proof is identical to that of proposition B.1.■

E Modeling Extension: Allowing for Signaling Prior to $T = 0$:

This section explores an extension in which one of the two countries has the ability to engage in sunk cost signaling before entering the war of attrition. That is, I consider an extension in which one country is granted the ability to "instanteously" produce large amounts of sunk cost signals prior to $t = 0$, mirroring costly signaling as in Fearon (1997), so that any learning that occurs from this signaling occurs before the war of attrition begins. I show that the allowing for such signaling does not eliminate the war of attrition - delay must still occur with positive probability. This demonstrates that the dynamic screening results are not an artificial consequence of a choice to prohibit states from engaging costly signaling in the classical sense.

Specifically, I show that when granted the ability to engage in sunk cost signaling in this way, there must be at least some types of $w_i \in [\underline{w}_i, 0)$ who will prefer to mimick any message sent by types $w_i \in [0, \overline{w}_i]$. This is because less resolved states' decision to invest more in sunk costs in the model is not driven by the game form but by their weaker incentive to spend resources to avoid war. However, so long as types $w_i < 0$ participate in the war of attrition, then both countries have some hope for a concession if they delay and Proposition (B. 1) and (B. 2) apply.

For simplicity's sake we will begin by considering the case where country i can send some costless message $m \in M$ where M is some compact metric space. We will define a signal as effective it induces all types of country j to exit immediately at the start of the war of attrition.

Definition D. 1 *A signal is an effective signal if in response to country i's message, types $w_j \leq 0$ concede at $t = 0$ and types $w_j > 0$ fight at $t = 0$.*

If a signal is effective, then best response of a resolved type of country i is to delay exit until after country j has exited since they incur no cost for delaying war and may obtain a concession. In effect, when a signal is effective, then the war of attrition is not played and the game ends immediately at $t = 0$.

The following result demonstrates that a signal can only be effective if it removes all doubt from country i's mind that country i is unresolved.

Lemma D. 1 *A costless or sunk cost signal m is an effective signal if and only if it induces a posterior belief that*

$$G_j(w_i|m) = \begin{cases} 0 \text{ if } w_i \leq 0 \\ \frac{f(w_i)}{1-F(0)} if w_i > 0 \end{cases} \tag{D. 1}$$

The intuition for this result is straightforward. If country i is resolved with certainty, then country j has no reason to delay exit and must do so immediately. However, if country i is possibly unresolved, then resolved types of country j have an incentive to delay and await its concession. By subgame perfection, any unresolved type of country i will use that opportunity to concede an arbitrarily short after time $t = 0$.

Note there can be a large number of signals that constitute an effective signal. For example, if there exist two messages m', m'' each sent only by types w_i', w_i'' respectively where $w_i', w_i'' \in (0, \overline{w}_i]$, then both will constitute effective signals. However, from the perspective of resolved types of country i the two messages are payoff equilivalent. It is without loss of generality to assume that an effective signal sent by country i therefore induces the beliefs listed in equation (D. 1).

It turns out that the countries cannot cause the war of attrition to end at time $t = 0$ and produce an effective signal with sunk cost signals.

Lemma D. 2 *No costless message can constitue an effective signal.*

Intuitively, this fails because there are types $w_i < 0$ who will become locked into conflict should they not send the effective signal and have to play the war of attrition game. These types are better off mimicking the signal rather than paying the costs of delay.

Now assume that messages are not costless and that sending a message $m \in \mathbb{R}^+$. Such that a country's expected utility from sending message is now given by equation (**??**) with an additional $-m$ for sending message m prior to the war of attrition. The following result demonstrates that this cannot prove an effective message.

Proposition D. 1 *No sunk cost message can constitue an effective signal.*

The intuition for this result, is similar to that for Lemma (D. 2). Types $w_i < 0$ who will become locked into conflict by sending an ineffective signal would be more likely to fight by sending the ineffective signal. These types are more incentivized to avoid war since they stand more to lose from fighting, therefore there exist types $w_i < 0$ would become locked into fighting under an alternate message who would prefer to spend more to produce an effective signal.

E.1 Proof of Lemma (D. 1)

We will begin by proving that a signal is effective only if it induces the posterior in equation (D. 1). That is, we will show that if Country j believes that it is possible that Country i may be a type $w_i < 0$, there will be unresolved and resolved types of country j remaining in the crisis after $t = 0$.

First, observe that if a signal is effective, then there is a unique equilibrium in which Country i chooses to go to war at some time $t > 0$, and all types of country j exit immediately. Resolved types of country j have no profitable deviation since they can expect no concession by delaying and will only pay additional sunk costs. Similarly unresolved types have no benefit to delay concession as they will only pay sunk costs and accumulate additional audience costs. Moreover, if they delay too long than Country i will fight. Finally, country i's specific choice of exit time is irrelevant so long as $t_i > 0$, since country i will exit before its choice is realized. Therefore, the only relevant deviation to consider is one in which Country i chooses to go to war at time $t = 0$, in which case it will have an expected utility of

$$U_i(0, 1|w_i) = F_j(0)\frac{1}{2}(1 + w_i) + (1 - F_j(0))w_i$$

which is strictly smaller than

$$U_i(t_i, 1|w_i) = F_j(0) + (1 - F_j(0))w_i$$

which is its expected utility for an exit time $t_i > 0$. This is sufficient to prove that a message that induces the belief in equation (D. 1) must be an effective signal.

Next, we will prove that if a message does not induce the beliefs in equation j, then it cannot be an effective signal and must have types of country j exit after $t = 0$. Suppose not. That is suppose that country j believed that there was some probability $z > 0$ such that country $i's$ type was $w_i < 0$. Moreover, suppose that all types of country j exited at time $t = 0$ either by going to war or conceding. As above, resolved types of country i with $w_i > 0$ would choose to exit by going to war after $t = 0$ and have no incentive to deviate from this action. Types $w_i < 0$ must make a choice. They can either choose to concede at time $t = 0$ for an expected utility of

$$U_i(0, 0|w_i) = \frac{1}{2}F_j(0) + (1 - F_j(0))\frac{w_i}{2}$$

or they can choose to exit after time $t > 0$ in exchange for a utility of

$$U_i(t_i, 1|w_i) = F_j(0) + (1 - F_j(0))w_i$$

Lt $\hat{w}_i$ denote the cutoff type that is indifferent between the two options. If there is a nondegnerate set of types $w_i < \hat{w}_i$ that prefers to concede at time $t = 0$. Then exiting at time $t = 0$ is no longer an equilibrium as all types of country j have a strictly profitable deviation to delaying exit by some arbitrarily small $\epsilon > 0$ and checking to see if country i concedes. This would be sufficient to prove that the message is not an effective signal in this instance.

Therefore, all that remains to show is that there will exist types of country j with an incentive to deviate and concede at a time $t > 0$ when they believe that there exist a nondegenerate set of types $w_i \in [\hat{w}_i, 0]$ in the war of attrition at time $t = 0$. First, observe that types of country i have selected a time to exit $t_i > 0$. If we require that i's strategies be subgame perfect, then this cannot be an equilibrium. Suppose that all resolved types of country j go to war at time t delay war until some arbitrary time $t_j > 0$ that may be arbitrarily small. Then there must exist some non-degenerate subset of types $w_i < 0$ who would prefer to concede immediately after $t = 0$, once a mass of types of country j have conceded. Therefore, this cannot be an equilibrium. ■

E.2 Proof of Lemma (D. 2)

Suppose not. That is, suppose that there existed a costless message that constituted an effective signal. The expected utility for type $w_i = 0$ from sending that signal is simply $F_j(0)$. Unresolved types of country i who do not send the effective message must either concede immediately or play a war of attrition game that goes past $t = 0$, per Lemma (D. 1). Note that there must exist a type $w_i = \epsilon$ for some ϵ that is arbitrarily small, that must get locked into fighting by audience costs in a war of attrition with positive length. This type's expected utility from participating in the war of attrition must be strictly less than from mimicking the costless signal since some types of country j will become locked into fighting as well and it must pay for the costs of delay. Thus all types in the interval $w_i \in [-\epsilon, 0)$ must also send the effective signal. A contradiction. ■

E.3 Proof of Proposition (D. 1)

Suppose that there existed a costly signal m^* that constituted an effective signal. Moreover, let m' denote the ineffective signal sent by a type $w_i = -\epsilon$ for some artbitarily small positive ϵ. Per lemma (D. 1), message m' must lead to a war of attrition with positive delay. Following the logic of Reich (2023) for this to be incentive compatible, the most type $w_i = 0$ would be willing to spend on the effective signal is given by

$$B(0) \equiv m^* - m' = F_j(0) - U_i(m'|0)$$

where $U_i(m'|0)$ is type $w_i = 0$ expected utility from sending signal m'. Note that because the equilibrium following signal m' has a higher probability of war since types $w_j < 0$ must be locked in by audience costs. It follows that type $B(-\epsilon)$, how much type $w_i = -\epsilon$ would be willing to spend on signal m^* instead of m' is strictly greater than $B(0)$. To see this simply observe that for a type who is going to become locked in by audience costs

$$\frac{U_i(m^*|w_i)}{\partial w_i} = 1 - F_j(0)$$

and that

$$\frac{U_i(m^*|0)}{\partial w_i} = 1 - F_j(\beta_j)$$

It follows that there must exist a type $w_i = -\epsilon$ who must strictly prefer to deviate from message m'_i to m^*, since they have a lower payoff to fighting and message m^* produces a lower probability of war than message m'.

References

Ashworth, Scott and Ethan Bueno de Mesquita (2006). "Monotone Comparative Statics for Models of Politics". *American Journal of Political Science* 50.1, pp. 214–231.

Fearon, James D. (1994). “Domestic Political Audiences and the Escalation of International Disputes”. *American Political Science Review* 88.3, pp. 577–592.

— (1997). “Signaling Foreign Policy Interests: Tying Hands versus Sinking Costs”. *Journal of Conflict Resolution* 41.1, pp. 68–90.

Hendricks, Ken, Andrew Weiss, and Charles Wilson (1988). “The War of Attrition in Continuous Time with Complete Information”. *International Economic Review* 29.4, pp. 663–680.

Reich, Noam (Mar. 2023). “When Can States Signal with Sunk Costs?” *Unpublished Manuscript*.

Takahashi, Yuya (2015). “Estimating a War of Attrition: The Case of the US Movie Theater Industry”. *American Economic Review* 105.7, pp. 2204–2241.